APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Scheme for Valuation/Answer Key

Scheme of evaluation (marks in brackets) and answers of problems/key

SECOND SEMESTER B.TECH DEGREE EXAMINATION -MAY, 2019

Course Code: MA102

Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks

1 Auxiliary equation $m^3 + 1 = 0$.......... (1 mark), 3

Roots $= -1, \frac{1\pm\sqrt{3}i}{2}$...............................(1 mark)

Writing solution $y = c_1 e^{-x} + e^{x/2}\left[c_2 \cos\frac{\sqrt{3}}{2}x + c_3 sin\frac{\sqrt{3}}{2}x\right]$..............(1 mark)

2 $\begin{vmatrix} e^x \cos 2x & e^x \sin 2x \\ e^x(\cos 2x - 2sin2x) & e^x(\sin 2x + 2\cos 2x) \end{vmatrix}$..................... (2 marks) 3

If Wronskian formula is there, give (1) mark

Evaluation and final answer $= 2e^{2x}$..(1 marks)

3 $PI = \frac{4}{D^2 - 4D - 5}(\cos 2x) = \frac{4}{-4D - 9}(\cos 2x) \ldots\ldots\ldots\ldots (1)$ 3

$= \frac{4}{145}(4\,D - 9)(\cos 2x) \ldots\ldots\ldots (1)$

$= \frac{-4}{145}(8 \sin 2x + 9 \cos 2x) \ldots\ldots\ldots\ldots (1)$

4 Calculating PI$= \frac{e^{2x}+e^{-2x}}{2[D^2+4D+4]} \ldots\ldots\ldots (1) = \frac{1}{2}[\frac{1}{16}e^{2x} - \frac{x^2}{2}e^{-2x}]$...(1+1 marks) 3

5 Calculating 3

$a_n = \frac{2}{\pi}\int_0^{\pi} \sin x Cos\, nx\, dx = \frac{2}{\pi(n^2-1)}[(-1)^{n+1} - 1], n \neq 1$...(2 marks)

$a_1 = 0$...(1 mark)

If only formula for a_n is there, give (1) mark

6 Calculating $b_n = \frac{2}{\pi}\int_0^{\pi} k \sin nx\, dx \ldots\ldots (1) = \frac{2k}{n\pi}[1 - (-1)^n]$...........(2 marks) 3

If b_n formula is there, give (1) mark

7 $x^2 + y^2 + (z - c)^2 = r^2 \ldots\ldots\ldots (1\, mark)$ 3

$x + (z - c)p = 0\ \&\ y + (z - c)q = 0 \ldots\ldots\ldots \quad (1\, mark)$

$qx - py = 0\ or\ \frac{x}{p} = \frac{y}{q} \ldots\ldots\ldots\ldots\ldots \quad (1 mark)$

8 $p = f'\left(\frac{xy}{z}\right)\left(\frac{zy-xyp}{z^2}\right) \dots \dots \dots (1\ mark) q = f'\left(\frac{xy}{z}\right)\left(\frac{zx-xyq}{z^2}\right)$......(1mark) 3

$px - qy = 0$.Or $\frac{p}{q}$.=.$\frac{z-xp}{y-zq}$.....(1 mark)

9 Arriving at equation $\frac{X'}{X} = \frac{4Y'}{Y}$...(1 mark), 3

$X = e^{kx+a}, Y = e^{\frac{ky}{4}+b} \dots \dots \dots (1) u(x,y) = 8e^{-3(4x+y)}$....(1 mark)

10 $y(0,t) = 0, y(l,t) = 0, \frac{\partial y}{\partial t} = 0\ at\ t = 0 \dots\dots\dots (1)$ 3

$y(x,0) = \frac{2hx}{l} \qquad 0 < x < \frac{l}{2}$

$= 2h(1 - \frac{x}{l}) \qquad \frac{l}{2} < x < l \dots\dots\dots \dots\dots (2)$

11 Steady state equation.........(1 mark), $u(0,t) = 0, u(30,t) = 80$ ...... (1) 3

Final answer $u = 2x + 20$.......(1)

12 $u(x,t) = \left(a\,e^{\lambda x} + be^{-\lambda x}\right)(ce^{\alpha^2\lambda^2 t})$ ...OR$\left(A\,e^{\lambda x} + Be^{-\lambda x}\right)(e^{\alpha^2\lambda^2 t})$(1 mark) 3

$u(x,t) = (a\cos\lambda x + b\sin\lambda x)\,ce^{-\alpha^2\lambda^2 t}OR(A\cos\lambda x + B\sin\lambda x)\,e^{-\alpha^2\lambda^2 t}$...(1 mark)

$u(x,t) = (ax + b)c$.OR $(Ax + B)$.................(1 mark)

PART B

Answer six questions,one full question from each module

Module 1

13 a) $m = -2 \pm i$ ....(1mark) $y = e^{-2x}(A\cos x + B\sin x) \dots \dots \dots (2\ marks)$ 6

$A = 2, b = -1$.........(2 marks)

$$y = e^{-2x}(2\cos x - \sin x) \dots \dots \dots (1\ mark)$$

b) Auxiliary equation$m^3 - m^2 + 4m = 0$ ... . (1 mark) 5

$m = 0, \frac{1}{2}(1 \pm i\sqrt{15})$........(2 marks)

$y = c_1 + e^{\frac{1}{2}x}(c_2\cos\frac{\sqrt{15}}{2}x + c_3\sin\frac{\sqrt{15}}{2}x)$......(2 marks)

OR

14 a) Writing $y_2 = vy_1 \dots \dots (1\ mark)$ 6

$\frac{du}{dx} = \frac{1}{y1^2}e^{-\int Pdx} = \frac{1+x^2}{x^2}$ ....(2 marks)

$u = x - \frac{1}{x}$................(2 mark)

$y_2 = x^2 - 1$...(1 mark)

b) $Auxilary\ equation\ (m^3 - 3m^2 - 4m + 6) = 0 \rightarrow (1), m = 1, 1 \pm \sqrt{7} \rightarrow (2)$ 5

$$y = c_1e^x + c_2e^{(1+\sqrt{(7)})\,x} + c_3e^{1-\sqrt{(7)}x} \rightarrow (3)$$

Module 1I

15 a) Substitution $3x + 1 = e^t$ ...(1 mark) 6

Conversion of given equation to $(2D^2 + 5D + 2)y = \frac{1}{3}(e^t - 1)$…(1 mark)

$$CF = ae^{-\frac{t}{2}} + be^{-2t} \ldots \ldots \ldots (1\ mark)$$

$PI = \frac{e^t}{27} - \frac{1}{6}$........(2 marks)

Final solution $y = c_1(3x+1)^{-\frac{1}{2}} + c_2(3x+1)^{-2} + \frac{1}{27}(3x+1) - \frac{1}{6}$…(1 mark)

b) $m = \pm i, \pm i$.......(1 mark) 5

CF= $(c_1 + c_2x)\cos x + (c_3 + c_4x)\sin x$…(1 mark)

Finding PI= $\frac{1}{(1+D^2)^2}x^4 \quad \ldots \ldots (1\ mark) = x^4 - 24x^2 + 72$…….(2 marks)

OR

16 a) Finding CF $= c_1\cos 2x + c_2\sin 2x$…(2 marks) 6

Wronskian $= 2$ …(1 mark)

Finding PI= $\frac{-\cos 2x}{4}[\log(\sec 2x + \tan 2x)]$ …(3 marks)

b) $m = 2, 2, CF = (c_1 + c_2x)e^{2x}$........(1 mark) 5

$PI = \frac{1}{2(D^2-4D+4)}(1 - \cos 2x)$........(1 mark)

$PI = \frac{1}{8} + \frac{Sin\ 2x}{16}$...........(2 marks)

Final solution $y = (c_1 + c_2x)e^{2x} + \frac{1}{8} + \frac{\sin 2x}{16}$…(1 mark)

Module 1II

17 a) Finding $a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)dx = 2$…(1 mark) 6

Finding $a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx = (-1)^{n+1}\frac{2}{n^2-1}, n \neq 1$…(1+2 marks)

Finding $a_1 = \frac{2}{\pi}\int_0^{\pi} f(x)\cos x\ dx = \frac{-1}{2}$ …(1 mark)

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$.....(1 mark)

b) Finding $a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)dx = \pi$…(1 mark) 5

Finding $a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx = \frac{2}{\pi n^2}[(-1)^n - 1]$…(2 marks)

Finding $b_n = 0$…(1 mark)

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$.....(1 mark)

OR

18	a)	Finding $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \pi$…(2 marks) Finding $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx = 0$ … (1 mark) Finding $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx = 0 \ldots \frac{1}{n}[1-(-1)^n]$.…(2 mark) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$.....(1 mark)	6
	b)	$a_0 = \frac{1}{2}\int_{-2}^{2} f(x)dx = k$…(1 mark) $a_n = \frac{1}{2}\int_{-2}^{2} f(x)\cos\frac{n\pi x}{2}dx = \left(\frac{2k}{n\pi}\right)\sin\left(\frac{n\pi}{2}\right)$ … (1mark)$b_n =$ $\frac{1}{2}\int_{-2}^{2} f(x)\sin\frac{n\pi x}{2}dx = 0$.…(2 mark) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{2} + b_n \sin\frac{n\pi x}{2}$…(1 mark)	5

Module 1V

19	a)	$\frac{xdx}{y^2z} = \frac{dy}{xz} = \frac{dz}{y^2}$… (2 Marks) $x^3 - y^3 = a$ &$x^2 - z^2 = b$ are two solutions.....(3 marks). $f(u,v) = 0$ or $u = f(v)$.......(1 mark) For any alternate correct answers give full marks	6
	b)	Equation of plane $lx + my + nz = 1$, where $l^2 + m^2 + n^2 = 1$ (2 Marks) Finding pde as $z = px + qy + \sqrt{p^2 + q^2 + 1}$…(3 Marks) OR Equation of plane $lx + my + nz + d = 0$......(1 mark) $\frac{\pm d}{\sqrt{l^2+m^2+n^2}} = k$.........(1 mark) $lx + my + nz \pm k\sqrt{l^2 + m^2 + n^2} = 0$......(1 mark) $l = -np \quad m = -nq$.........(1 mark) $z = px + qy \pm k\sqrt{p^2 + q^2 + 1}$…(1 marks)	5

OR

20	a)	$$\frac{dx}{x^2(y-z)} = \frac{dy}{y^2(z-x)} = \frac{dz}{z^2(x-y)} \ldots\ldots (2\, marks)$$ $xyz = a$ &$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = b$ are two solutions.......(3 marks) $f(u,v) = 0$ or $u = f(v)$.......(1 mark) For any alternate correct answers give full marks	6
	b)	Finding characteristic roots $m = -2, -1$.…(1 Mark)	5

$CF = \phi_1(y - x) + \phi_2(y - 2x)$….(1 Mark)

Evaluating $PI = \frac{x^4y^2}{12} - \frac{x^5y}{10} + \frac{7x^6}{180}$ …(3 Marks)

Module V

21 Wave equation, Initial and Boundary conditions…(3 Marks) 10

General solution

$u(x,t) = (A\cos\lambda x + B\sin\lambda x)(C\cos a\lambda t + D\sin at)$ ....(1 Mark)

A = 0, $\lambda = \frac{n\pi}{60}$ .... (2)

A_n=0 ….(1 mark)

Evaluating $b_n = \frac{480\lambda}{n^3\pi^3 a}\sin\frac{n\pi}{2}$…(3 marks)

OR

22 Wave equation, Initial and Boundary conditions…(3 Marks) 10

$u(x,t) = (A\cos\lambda x + B\sin\lambda x)(C\cos a\lambda t + D\sin at)$ ....(1 Mark)

A = 0, $\lambda = \frac{n\pi}{60}$ .... (2)

$b_n = 0$.........(1)

Evaluating $A_n = \frac{14400}{n^3\pi^3}\{1-(-1)^n\}$ …(3 Marks)

Module VI

23 Heat equation, Initial and Boundary conditions….......(3 Marks) 10

General solution$u(x,t) = a\cos\lambda x + b\sin\lambda x)\, ce^{-\alpha^2\lambda^2 t}$ … … . . (1 mark)

A = 0, $\lambda = \frac{n\pi}{2}$ … .... (2)

Evaluating $B_n = \frac{1600}{n^3\pi^3}\{1-(-1)^n\}$ ….(3 Marks)

Writing final solution …(1 Marks)

OR

24 Steady state conditions $u = 5x + 50$ …(2 Marks) 10

Initial and boundary conditions …..(2 Marks)

$u(x,t) = u_1(x) + u_2(x,t)$ ….(1 Mark) $u_1(x) = -3x + 90$………(2 Marks)

Evaluating $u_2(x,t) = \sum_{n=1}^{\infty}\frac{-80}{n\pi}[(-1)^n + 1]\sin(\frac{n\pi x}{10})\, e^{\frac{-\alpha^2 n^2\pi^2 t}{100}}$ …. (2 Marks)

Writing final $u(x,t)$….(1 Mark)

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