

# APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

## Scheme of Valuation/Answer Key

*Scheme of evaluation (marks in brackets) and answers of problems/key*

**SEVENTH SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2018**

**Course Code:AE405**

**Course Name: ADVANCED CONTROL THEORY**

Max. Marks: 100

Duration: 3 Hours

### PART A

*Answer any two full questions, each carries 15 marks.*

Marks

- 1 a) 1 mark for each (5)
- b) Statespace 8mark, statediagram 2mark (10)

I

b.  $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y(t) = r(t)$

$x_1 = y$  ;  $\dot{x}_1 = x_2$

$x_2 = \frac{dy}{dt}$  ;  $\dot{x}_2 = \frac{dy}{dt} = -6\frac{dy}{dt} - 8y + r(t)$

$\dot{x}_2 = -8y - 6\frac{dy}{dt} + r(t)$

$\dot{x}_2 = -8x_1 - 6x_2 + r$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} r_1 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

- 2 a) Singularity 2 mark, type 3 mark (5)
- b) 10 marks (10)
- 3 a) Singularity 3marks , Plot of trajectory 7 marks (10)
- b) State transition matrix 5 marks (5)

### PART B

*Answer any two full questions, each carries 15 marks.*

4 a) Describing function of relay 5 marks (5)

b) Now, 
$$G(j\omega) = \frac{1}{j\omega(1+j\omega)(2+j\omega)} = \frac{(1-j\omega)(2-j\omega)}{j\omega(1+\omega^2)(4+\omega^2)} = \frac{2-j3\omega-\omega^2}{j\omega(1+\omega^2)(4+\omega^2)} \quad (10)$$

Separating  $G(j\omega)$  into real and imaginary parts, we obtain

$$\text{Im } G(j\omega) = \frac{2-\omega^2}{j\omega(1+\omega^2)\omega(4+\omega^2)} = 0$$

or 
$$\omega^2 = 2 \Rightarrow \omega = \pm\sqrt{2}$$

Now, 
$$\text{Re } G(j\omega) = \frac{-j3\omega}{j\omega(1+\omega^2)(4+\omega^2)} = \frac{-3}{(1+\omega^2)(4+\omega^2)} = \frac{-3}{(1+2)(4+2)} = -\frac{3}{18} = -\frac{1}{6}$$

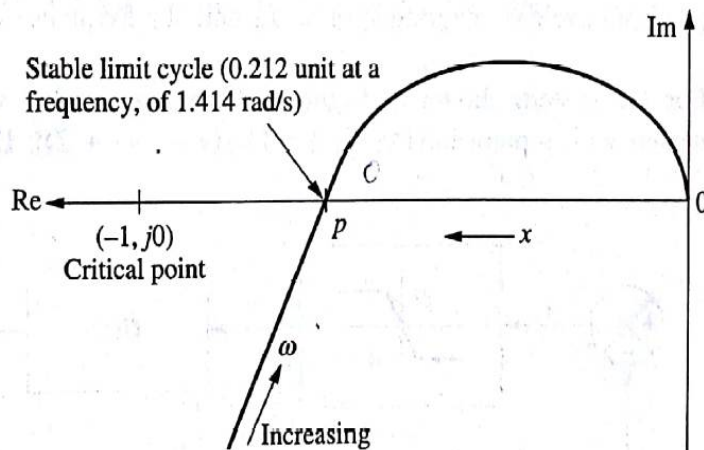
Therefore, at the point of intersection

$$\frac{1}{k_{eq}} = -\frac{\pi E}{4} = -\frac{1}{6}$$

$\therefore E = \frac{4}{6\pi} = 0.212$  unit and frequency = 1.414 rad/s.

So, the coordinate of  $P$  is  $(-0.212, j0)$  and is very much within the critical point  $(-1, j0)$ .

Therefore, a stable limit cycle of magnitude of 0.212 unit at a frequency of 1.414 rad/s exists.



5 a) Stability 5 mark, lyapunov function 5 marks (10)

b) Sign definiteness with eg 5 mark (5)

6 a) 2.5 marks each comparison (5)

b) Explanation with diagram 10 marks (10)

**PART C**

*Answer any two full questions, each carries 20 marks.*

- 7 a) Observability matrix 6 marks, checking Rank same or not 3 marks, comment on observability 1 mark (10)
- b) 5 marks Pulse transfer function (5)
- c) Explanation 3 mark, diagram of mapping 2 marks (5)
- 8 a) 15 marks (15)
- b) ROC 2 marks, properties 3 marks (5)
- 9 a) Controllability matrix 6 marks, Checking Rank same or not 3 marks, comment on controllability 1 mark (10)
- b) 10 marks (10)

$$9 \quad (b) \quad x(k+2) + 3x(k+1) + 2x(k) = 0 ; \quad x(0) = 0 ; \quad x(1) =$$

$$z(x(k+2)) = z^2 X(z) - z^2 x(0) - z x(1)$$

$$z(x(k+1)) = z X(z) - z x(0)$$

$$z(x(k)) = X(z)$$

Applying this in eq 9

$$z^2 X(z) - z^2 x(0) - z x(1) + 3z(x(0)) + 2(X(z)) = 0$$

Substituting the initial data & simplifying gives.

$$X(z) = \frac{z}{z^2 + 3z + 2} = \frac{z}{(z+1)(z+2)} = \frac{z}{z+1} - \frac{z}{z+2}$$

$$x(k) = (-1)^k - (-2)^k ; \quad k = 0, 1, 2, \dots$$

\*\*\*\*