Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

V SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2018

Course Code: CS309

Course Name: GRAPH THEORY AND COMBINATORICS

Max. Marks: 100 Duration: 3 Hours

PART A

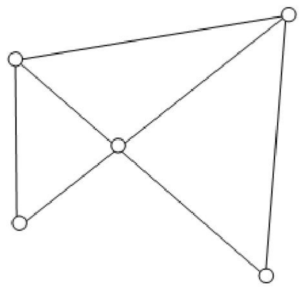
Answer all questions, each carries 3 marks. Marks

1 Prove that the number of vertices of odd degree in a graph is always even (3)

2 Show that in a simple graph with n vertices, the maximum number of edges is n(n-1)/2 and the maximum degree of any vertex is n-1. (3)

3 Differentiate between complete symmetric and complete asymmetric graph with an example each. (3)

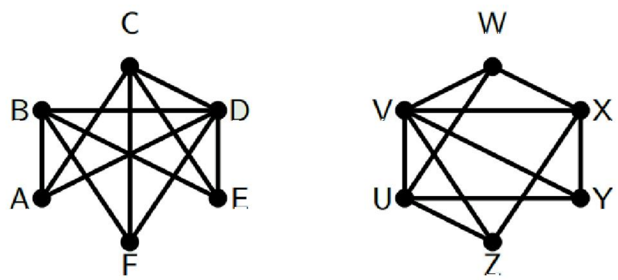
4 State Dirac's Theorem and check its applicability in the following graph, G (3)



PART B

Answer any two full questions, each carries9 marks.

5 a) Define isomorphism between graphs? Are the two graphs below isomorphic? Justify (5)



b) Consider a complete graph G with 11 vertices. (4)

1. Find the maximum number of edges possible in G.
2. Find the number of edge-disjoint Hamiltonian circuits in G.

6 a) A connected graph G is an Euler graph if and only if all vertices of G are of even degree.Prove the statement. (6)

b) There are 37 telephones in the city of Istanbul, Turkey. Is it possible to connect them with wires so that each telephone is connected with exactly 7 others?Substantiate your answer with graph concepts. (3)

7 a) Give any two applications of graphs. Explain. (2)

b) Define Hamiltonian circuit. Give an example. What general class of graphs is guaranteed to have a Hamiltonian circuit? Also draw a graph that has a Hamiltonian path but not a Hamiltonian circuit. (4)

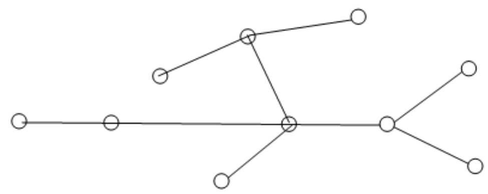
c) Prove that if a connected graph G is decomposed into two subgraphs g1 and g2 , there must be at least one vertex common between g1 and g2 (3)

PART C

Answer all questions, each carries 3 marks.

8 Prove that the distance between vertices of a connected graph is a metric. (3)

9 i) Find the eccentricity of all vertices in G given below and also mark the center of G (3)



ii) Find the number of possible labelled trees that can be constructed with 50 vertices.

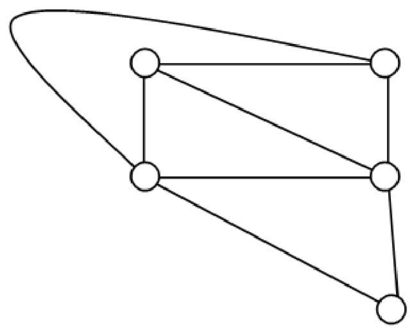
10 Draw the two simplest non-planar graphs and also mention their properties. (3)

11 What is the necessary and sufficient condition for two graphs to be duals of each other? Prove. (3)

PART D

Answer any two full questions, each carries 9 marks.

12 a) Draw the geometric dual (G*) of G given and also write about the relationship between a planar graph G and its dual G* (6)



b) Define rooted binary tree with an example (3)

13 a) Find the number of edges and vertices of a graph G if its rank and nullity are 6 and 8 respectively (2)

b) Prove the statement,"Every circuit has an even number of edges in common with any cut-set" (4)

c) Consider a binary tree with four weighted pendant vertices. Let their weights be0.5, 0.12, 0.3 and 0.11. Construct a binary tree with minimum weighted path length. (3)

14 a) Define cut sets with an example. Give an application of finding cut-sets or edge (4)

connectivity

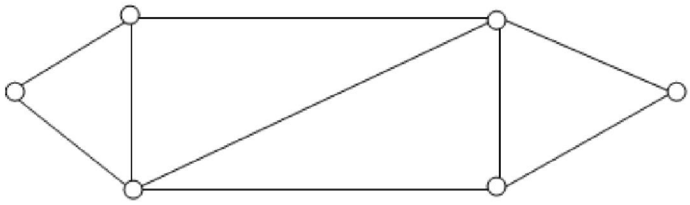
b) Define spanning tree. Show that the edges forming a spanning tree in a planar graph G correspond to the edges forming a set of chords in the dual G* (5)

PART E

Answer any four full questions, each carries 10 marks.

15 a) Draw the flow chart of spanning tree algorithm and also clearly mark the five conditions to be tested in connection with the spanning tree construction in the flowchart (6)

b) Obtain a cut-set matrix for the following graph: (4)



16 a) Draw the flowchart to determine the components of a graph. (6)

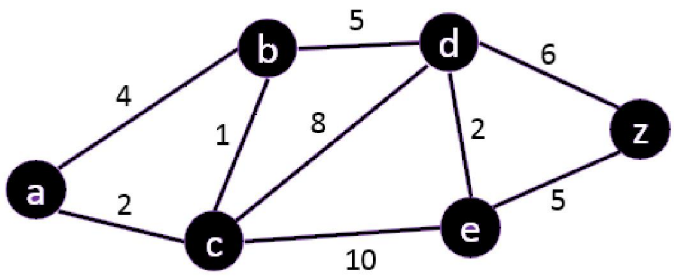
b) Define adjacency matrix and construct a graph from the following adjacency matrix: (4)

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

17 a) Write edge listing and successor listing methods used in computer representation of graphs. (4)

b) Two graphs G1 and G2 are isomorphic if and only if their incidence matrices A(G1) and A(G2) differ only by permutations of rows and columns (6)

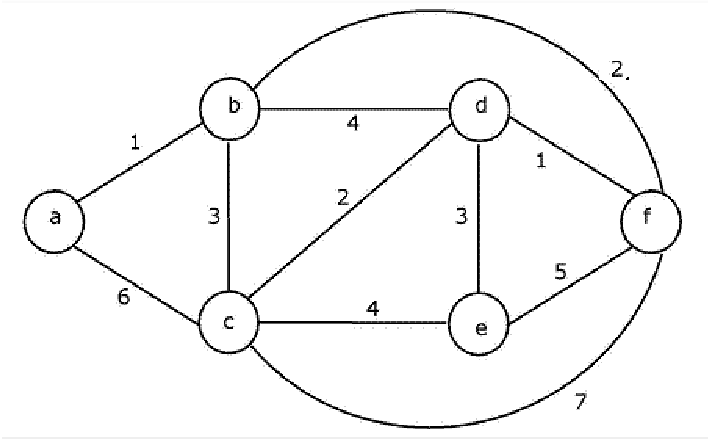
18 a) Write the Dijkstra's Shortest Path Algorithm and apply this algorithm to find the shortest path between a and z (6)



b) Let A and B be, respectively, the circuit matrix and incidence matrix of a self-loop-free graph G. Prove that $A \times B^T = 0 \pmod 2$ (4)

19 a) Define cut-set matrix and list down any four properties of cut-set matrix (5)

b) Apply Kruskal's procedure to find the minimum spanning tree from the (5) following graph G.



20 a) Prove that if B is a circuit matrix of a connected graph G with e edges and n vertices , then rank of B = e-n+1 (5)

b) How can two linear arrays be used to represent a digraph. Give an example. Compare this representation with edge list representation in terms of storage. (5)

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