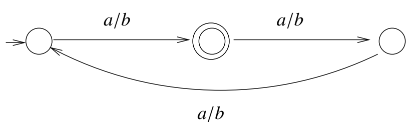
Name: ....
Roll No.: ....

M.Tech. Degree Examination - December 2015
APJ Abdul Kalam Technological University - Cluster No. 04
Branch/Specialisation : CSE/Computer Science and Systems Engineering
Subject : 04 CS 6405 - Automata Theory and Computability

Time: 03 Hrs   Max. Marks: 60

Part A (Answer All, Each Carries 03 Marks)

1. What language is represented by the following DFA ?



2. Design a DFA which accepts precisely the set of all binary strings having odd number of 0s and odd number of 1s.
3. What language is represented by the regular expression: $b(a+b)^* + (b+a)^*a$?
4. Give a regular expression for the language: $\mathcal{L} = \{w \in \{a,b\}^* \mid w$ does not contain consecutive as$\}$.
5. What language is represented by the MSO sentence: $\forall x\ ((\neg last(x) \wedge Q_b(x) \wedge succ(x,y)) \implies Q_a(y))$.
6. How many equivalence classes are there in the canonical Myhill Nerode relation for the language $\mathcal{L} = \{w \in \{a,b\}^* \mid \text{ length}(w) \text{ is a multiple of } 3\}$
7. Argue that condition in the Parikh's theorem is not sufficient for proving that a language is context-free.
8. Let $\mathcal{L}$ be a non-recursive language recognised by a Turing Machine. Then what you can say about the complement of $\mathcal{L}$?

[08 X 03 = 24 Marks]

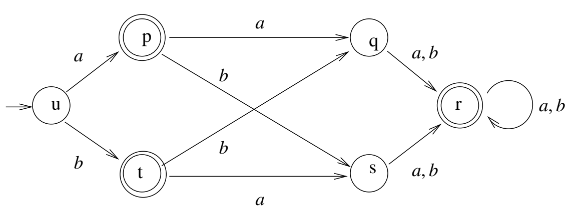
Part B (Answer All, Each Carries 06 Marks)

[06 X 06 = 36 Marks]

9. Give an NFA for the language $\mathcal{L} = \{0w1 \mid w \in \{0,1\}^*\}$. Also apply subset construction on the NFA to obtain the equivalent DFA.

OR

10. Obtain the unique-minimal DFA corresponding to the canonical MN relation representing the language recognised by the following DFA.



11. Give an MSO sentence representing the language $\mathcal{L} = \{w \in \{a,b\}^* \mid w$ contains the substring $abb\}$.

OR

12. Give an MSO sentence representing the language over the alphabet set $\{a,b\}$, where each string in the language contains odd number of bs.

13. Prove that the language $\mathcal{L} = \{a^n \mid \text{n is prime}\}$ is not regular.

OR

14. Using MN Theorem argue that the language $\mathcal{L} = \{a^{3*n} \mid n \geq 0\}$ is regular.

15. Give a Context-Free Grammar (CFG) for the language:
$\mathcal{L} = \{a^m b^n \mid n > m\}$.

OR

16. Give a PDA (accepts by emptying stack) accepting the language $\mathcal{L} = \{w \in \{a, b\}^* \mid \#_a(w) = 2 * \#_b(w)\}$, where the notation $\#_0(w)$ represents the number of zeros present in w. (No explanation is required. It is enough to give the set of transitions or transition graph where a transition $\delta(p, a, X) = (q, YX)$ is represented by the edge $\text{ⓟ} \xrightarrow{(a,X)/YX} \text{ⓠ}$.)

17. Design a Turing Machine for checking whether a number m is greater than or equal to a number n. Assume that the tape initially contains the unary representations of m and n separated by the symbol \$. That is the initial tape content will be $\vdash 1^m \$ 1^n \flat^\omega$, where $\flat$ represents the blank symbol. Design the Turing Machine to halt in the state t if $m \geq n$ and to halt in the state r if $m < n$. (No explanation is required. It is enough to give the set of transitions or transition graph where a transition $\delta(p, a) = (q, b, R)$ is represented by the edge $\text{ⓟ} \xrightarrow{a/(b,R)} \text{ⓠ}$.

OR

18. Design a Turing Machine to accept the language $\mathcal{L} = \{w \in \{a, b\}^* \mid \text{length}(w) \text{ is even}\}$. (No explanation is required. It is enough to give the set of transitions or transition graph where a transition $\delta(p, a) = (q, b, R)$ is represented by the edge $\text{ⓟ} \xrightarrow{a/(b,R)} \text{ⓠ}$.)

19. Show without using Rice's Theorem that the language $CFL = \{M \mid \mathcal{L}(M) \text{ is context-free}\}$ is not decidable.

OR

20. State and prove Rice's second theorem.