Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION,DECEMBER 2018

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks. Marks

1	a)	Test the convergence of $\sum_{k=1}^{\infty}\frac{\cos k}{k^2}$	(2)
	b)	Discuss the convergence of $\sum_{k=1}^{\infty}\frac{(2k)!}{4^k}$	(3)
2	a)	Find the slope of the surface $z=\sin(y^2-4x)$ in the $x-$ direction at the point $(3,1)$.	(2)
	b)	Find the differential dz of the function $z=\tan^{-1}(x^2y)$.	(3)
3	a)	Find the direction in which the function $f(x,y)=xe^y$ decreases fastest at the point $(2,0)$.	(2)
	b)	Find the tangent plane to the elliptic paraboloid $z=2x^2+y^2$ at $(1,1,3)$	(3)
4	a)	Evaluate $\iint_R y\sin xy\ dA$, where $\boldsymbol{R}=[1,2]\times[0,\pi]$.	(2)
	b)	Evaluate $\int_0^2\int_0^1\frac{x}{(1+xy)^2}$ dy dx	(3)
5	a)	if $\vec{A}=(3x^2+6y)\boldsymbol{i}-14yz\boldsymbol{j}+20xz^2\boldsymbol{k}$,evaluate $\int\vec{A}.d\vec{r}$ from (0,0,0) to(1,1,1) along the path, $x=t, y=t^2, z=t^3$	(2)
	b)	Prove that $\vec{F}=(x^2-yz)\boldsymbol{i}+(y^2-xz)\boldsymbol{j}+(z^2-xy)\boldsymbol{k}$ is irrotational.	(3)
6	a)	Determine the source and sink of the vector field $\boldsymbol{F}(x,y,z)=2(x^3-2x)\boldsymbol{i}+2(y^3-2y)\boldsymbol{j}+2(z^3-2z)\boldsymbol{k}$	(2)
	b)	Evaluate $\iint_S\vec{F}.\vec{n}ds$ where S is the surface of the cylinder $x^2+y^2=4$, $z=0$, $z=3$ where $\vec{F}=(2x-y)\bar{i}+(2y-z)\bar{j}+z^2\bar{k}$	(3)

PART B

Module 1

Answer any two questions, each carries 5 marks.

7		Check the convergence of the series $\frac{3}{4}+\frac{3.4}{4.6}+\frac{3.4.5}{4.6.8}+\frac{3.4.5.6}{4.6.8.10}+\cdots$	(5)
8		Find the radius of convergence of the power series $\sum_{k=1}^{\infty}\frac{(-1)^k(x-4)^k}{3^k}$	(5)
9		Determine whether the alternating series $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{3^{2k-1}}{k^2+1}$ is absolutely	(5)

convergent.

Module 1I

Answer any two questions, each carries 5 marks.

10 If $u = x^2 tan^{-1}\left(\frac{y}{x}\right) - y^2 tan^{-1}\left(\frac{x}{y}\right)$, find $\frac{\partial^2 u}{\partial x \partial y}$ (5)

11 Let $z = xye^{\frac{x}{y}}, x = r\cos\theta,\ y = r\sin\theta$, use chain rule to evaluate $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$ at $r = 2$ and $\theta = \frac{\pi}{6}$ (5)

12 A rectangular box open at the top is to have volume $32m^3$. Find the dimensions of the box requiring least material for its construction. (5)

Module III

Answer any two questions, each carries 5 marks.

13 Suppose that a particle moves along a circular helix in 3-space so that its position vector at time **t** is **r**(t) = $4\cos\pi t\,\mathbf{i} + 4\sin\pi t\,\mathbf{j} + t\,\boldsymbol{k}$. Find the distance travelled and the displacement of the particle during the time interval $1 \le t \le 5$. (5)

14 Suppose that the position vector of a particle moving in a plane $\bar{r} = 12\sqrt{t}\,\boldsymbol{i} + t^{\frac{3}{2}}\boldsymbol{j}, t > 0$. Find the minimum speed of the particle and locate when it has minimum speed? (5)

15 Find the parametric equation of the tangent line to the curve $x = \cos t, y = \sin t, z = t$ where t = t_0 and use this result to find the parametric equation of the tangent line to the point where t = π. (5)

Module 1V

Answer any two questions, each carries 5 marks.

16 Evaluate $\iint_R y\,dA$ where R is the region in the first quadrant enclosed between the circle $x^2 + y^2 = 25$ and the line $x + y = 5$. (5)

17 $Evaluate \int_1^2 \int_0^x \frac{dy\,dx}{x^2 + y^2}$ (5)

18 Evaluate $\iiint_V x\,dxdydz$ where V is the volume of the tetrahedron bounded by the plane $x = 0, y = 0, z = 0\ x + y + z = a.$ (5)

Module V

Answer any three questions, each carries 5 marks.

19 Find the scalar potential of $\vec{F} = (2xy + z^{3)}\boldsymbol{i} + x^2\boldsymbol{j} + 3xz^2\,\boldsymbol{k}$ (5)

20 Find the work done by $F(x, y) = (x^2 + y^2)\boldsymbol{i} - x\boldsymbol{j}$ along the curve $C: x^2 + y^2 = 1$ counter clockwise from $(1,0)$ to $(0,1)$. (5)

21 Evaluate $\int_C \bar{F}\,.\,d\bar{r}$ where $\bar{F} = y^2\boldsymbol{i} + xy\,\boldsymbol{j}$ and $\bar{\boldsymbol{r}}(t) = t\boldsymbol{i} + 2t\boldsymbol{j}$, $1 \le t \le 3$. (5)

22 Evaluate $\int\ y\,dx + z\,dy + x\,dz$ along the path $x = \cos \pi t, y = \sin \pi t\ ,\ z = t$ from $(1,0,0)$ to $(-\mathbf{1},0,1)$ (5)

23 If $\bar{r} = x\,\bar{\imath} + y\,\bar{\jmath} + z\,\bar{k}$ and $= \|\bar{r}\|$, prove that $\nabla^2 f(r) = \frac{2}{r} f'(r) + f''(r)$. (5)

Module VI

Answer any three questions, each carries 5 marks.

24 Using Stoke's theorem evaluate $\int_C \overline{F}\,.\,d\bar{r}$; where $\bar{F} = xy\,\bar{\imath} + yz\,\bar{\jmath} + xz\,\bar{k}$; C triangular path in the plane $x + y + z = 1$ with vertices at $(1,0,0), (0,1,0)$and (0,0,1) in the first octant (5)

25 Using Green's theorem evaluate $\int_C (y^2 - 7y)dx + (2xy + 2x)\,dy$ where C is the circle $x^2 + y^2 = 1$ (5)

26 Find the mass of the lamina that is the portion of the cone $z = \sqrt{x^2 + y^2}$ between $z = 1$ and $z = 3$ if the density is $\emptyset(x, y, z) = x^2 z$. (5)

27 Use divergence theorem to find the outward flux of the vector field $F(x, y, z) = x^3 i + y^3 j + z^3 k$ across the surface σ bounded by $x^2 + y^2 = 4,\ \ z = 0$ and $z = 4$. (5)

28 If S is the surface of the sphere $x^2 + y^2 + z^2 = \mathbf{1}$,Evaluate

$$\iint_s (xi + 2yj + 3zk)\,.\,dS$$ (5)

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