Reg No.:_____   Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER B.TECH DEGREE EXAMINATION (S), MAY 2019
Course Code: MA101
Course Name: CALCULUS

Max. Marks: 100   Duration: 3 Hours

PART A
Answer all questions, each carries 5 marks.   Marks

1 a) Check the convergence of the series $\sum_{k=1}^{\infty}\left(\frac{3k-4}{4k-5}\right)^k$ (2)

   b) Find the Maclaurin series of $f(x)=\frac{1}{1+x}$, up to 3 terms   (3)

2 a) If $z=(3x-2y)^4$, find $\frac{\partial^4 z}{\partial x\,\partial y^3}$ (2)

   b) If $w=\log(\tan x+\tan y+\tan z)$ then prove that   (3)
       $$\sin 2x\frac{\partial w}{\partial x}+\sin 2y\frac{\partial w}{\partial y}+\sin 2z\frac{\partial w}{\partial y}=2$$

3 a) Find the speed of a particle moving along the path $x=2\cos t, y=2\sin t, z=t$ (2)

       at $t=\pi/2$

   b) If $y'(t)=\cos t\,i+\sin t\,j$; $y(0)=i-j$. Find $y(t)$.   (3)

4 a) Evaluate $\int_0^1\int_0^{x^2}\int_0^2 dy\,dz\,dx$ (2)

   b) Evaluate $\iint xy\,dx\,dy$ over the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and lying   (3)

       in the first quadrant.

5 a) Show that $F(x,y)=2xy^3 i+3x^2y^2 j$ is conservative.   (2)

   b) If $\bar{r}=x\bar{i}+y\bar{j}+z\bar{k}$ and $r=\|\bar{r}\|$, prove that $\nabla.\frac{\bar{r}}{r^3}=0$ (3)

6 a) Evaluate by Stoke's theorem $\oint_C (e^x\,dx+2y\,dy-dz)$, where C is the curve   (2)
       $x^2+y^2=4, z=2$

   b) Using Green's theorem evaluate $\int_C x\,dy-y\,dx$ where C is the circle $x^2+y^2=4$ (3)

PART B
Module 1
Answer any two questions, each carries 5 marks.

7 Test for convergence of the series $\sum_{k=1}^{\infty}\frac{1}{(8k^2-3k)^{1/3}}$ (5)

8 Find the radius of convergence and interval of convergence of the power   (5)

   series $\sum_{k=0}^{\infty}\frac{(2x-1)^k}{3^{2k}}$.

9
Show that the series $\sum_{k=1}^{\infty}(-1)^k \left(\frac{k}{k+1}\right)^{k^2}$ is convergent.   (5)

Module 1I
Answer any two questions, each carries 5 marks.

10   Let $w = \sqrt{x^2 + y^2 + z^2}$, where $x - \cos\theta, y - \sin\theta, z - \tan\theta$. Find $\frac{dw}{d\theta}$ at   (5)

$\theta = \frac{\pi}{4}$, using chain rule.

11   Find the local linear approximation $L(x,y)$ to $f(x,y) = \ln(xy)$ at the point   (5)

$P(1,2)$. Compare the error in approximating f by L at the point $Q(1.01,2.01)$

with the distance between P and Q.

12   Find relative extrema and saddle points, if any, of the function   (5)

$f(x,y) = xy + \frac{8}{x} + \frac{8}{y}$.

Module 1II
Answer any two questions, each carries 5 marks.

13   Find the unit tangent T(t) and unit normal N(t) to the curve   (5)

$x = a\cos t, \ y = a\sin t, \ z = ct \qquad a > 0$

14   Find the velocity and position vectors of the particle ,if the acceleration vector   (5)

$a(t) = \sin t i + \cos t j + e^t k \ ; \ v(0) = k; \ r(0) = -i + k$.

15   Find the equation of the tangent line to the curve of intersection of surfaces   (5)

$z = x^2 + y^2$ and $3x^2 + 2y^2 + z^2 = 9$ and the point $(1,1,2)$.

Module 1V
Answer any two questions, each carries 5 marks.

16
Evaluate by reversing the order of integration $\int\limits_{0}^{a/\sqrt{2}} \int\limits_{y}^{\sqrt{a^2-y^2}} x \, dx \, dy$ (5)

17   Evaluate $\iint\limits_{R} xy \, dA$, where R is the sector in the first quadrant bounded by   (5)

$y = \sqrt{x}, \ y = 6 - x, \ y = 0$.

18   Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dz \, dx \, dy$ (5)

Module V

Answer any three questions, each carries 5 marks.

19   Find the work done by $F(x,y) = (x^2 + y^2)i - xj$ along the curve   (5)

$C: x^2 + y^2 = 1$ counter clockwise from $(1,0)$ to $(0,1)$

20   Determine whether $F(x,y) = 6y^2 i + 12xy j$ is a conservative vector field. If   (5) so find the potential function for it.

21   Find the divergence and curl of the vector field   (5)

$F(x,y,z) = xyz^2 i + yzx^2 j + zxy^2 k$

22   Prove that $\int_C (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k} \cdot d\bar{r}$ is independent of the path   (5)

and evaluate the integral along any curve from (0,0,0) to (1,2,3).

23   If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ and $r = \|\bar{r}\|$, prove that $\nabla^2 f(r) = \frac{2}{r} f'(r) + f''(r)$.   (5)

Module VI

Answer any three questions, each carries 5 marks.

24   Using Green's theorem evaluate $\int_C (xy + y^2)dx + x^2 dy$ where C is the boundary   (5)
of the region bounded by $y = x^2$ and $x = y^2$

25   Evaluate the surface integral $\iint_\sigma z^2 ds$, where σ is the portion of the curve   (5)
$z = \sqrt{x^2 + y^2}$ between $z = 1$ and $z = 3$

26   Determine whether the vector field $F(x,y,z)$ is free of sources and sinks. If not,   (5)
locate them.

     i) $F(x,y,z) = (y + z)\bar{i} - xz\bar{j} + x^2 \sin y \, \bar{k}$
     ii) $F(x,y,z) = x^3\bar{i} + y^3\bar{j} + 2z^3\bar{k}$

27   Use divergence theorem to find the outward flux of the vector field   (5)

$F(x,y,z) = (2x + y^2)i + xy j + (xy - 2z)k$ across the surface σ of the

tetrahedron bounded by $x + y + z = 2$ and the coordinate planes.

28   Using Stoke's theorem evaluate $\int_C \bar{F} \cdot d\bar{r}$; where $\bar{F} = xy\bar{i} + yz\bar{j} + xz\bar{k}$;   (5)

C triangular path in the plane $x + y + z = 1$ with vertices at $(1,0,0), (0,1,0)$ and

(0,0,1) in the first octant

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