APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M. TECH DEGREE EXAMINATION

Civil Engineering

(Structural Engineering and Construction Management)

04 CE 6401 – Analytical Methods in Engineering

Max. Marks : 60 Duration: 3 Hours

PART A

Answer All Questions

Each question carries 3 marks

1. Solve $y'' + y = 0$, given $y(0) = 2$, $y(\pi) = -2$
2. Solve $x(y-z)p + y(z-x)q = z(x-y)$
3. Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x}$
4. Using method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x,0) = 6e^{-3x}$
5. Classify the equation $(1+x^2)\frac{\partial^2 u}{\partial x^2} + (5+2x^2)\frac{\partial^2 u}{\partial x \partial t} + (4+x^2)\frac{\partial^2 u}{\partial t^2} = 0$
6. Classify the equation $y^2 u_{xx} - 2y\, u_{xy} + u_{yy} - u_y = 8y$
7. Derive Standard 5 point formula
8. The transverse displacement u of a point at a distance x from one end and at any time t of a vibrating string satisfying the equation $u_{tt} = 4u_{xx}$, with the boundary conditions $u = 0$ at x=0,t>0 and u=0 at x=4,t>0 and initial conditions $u = x(4-x)$ and $\frac{\partial u}{\partial t} = 0$ at $t = 0$, $0 \le x \le 4$. Solve this equation for one half period of vibration, taking $h = 1$ and $k = \frac{1}{2}$.

PART B

Each question carries 6 marks

9. By the method of variation of parameters, solve $(D^2 - 2D + 1)y = e^x \log x$

OR

10. Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$

11. Show that the equations $xp - yq = 0$, $z(xp + yq) = 2xy$ are compatible and solve them.

OR

12. Find the surface which intersects the surface of the system $z(x+y) = c(3z+1)$ orthogonally which passes through the circle $x^2 + y^2 = 1 \; and \; z = 1$

13. Solve by Charpit's Method $(p^2 + q^2)y = qz$

OR

14. Solve $4\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 16 \log(x + 2y)$
15. A string is stretched and fastened to 2 parts l apart. Motion is started by displacing the string in the form $y = a \sin\frac{\pi x}{l}$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by $y(x,t) = a \sin\frac{\pi x}{l}\cos\frac{\pi ct}{l}$

OR

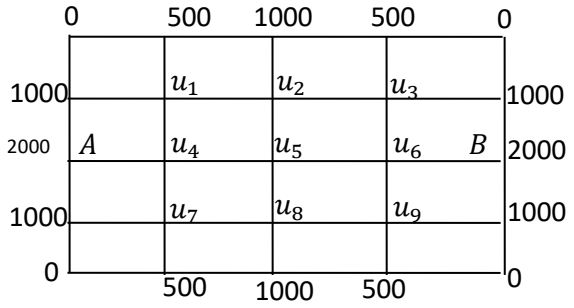
16. Derive solutions of Laplace's Equation of 2 dimensions

17. In which part of the XY Plane is the following equation elliptic $u_{xx}+u_{xy}+(x^2+4y^2)u_{yy}=2\sin(xy)$

OR

18. Classify the equation $x^2\, u_{xx} + (1-y^2)\, u_{yy} = 0$ where $-\infty < x < \infty$ and $-1 < y < 1$

19. Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary values as shown



OR

20. Evaluate the pivotal values of the equation $u_{tt} = 16u_{xx}$, taking $h = 1$ upto $= 1.25$. The boundary conditions are $u(0,t) = u(5,t) = 0$, $u_i(x,0) = 0$ and $u(x,0) = x^2(5-x)$