

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH DEGREE EXAMINATION

Civil Engineering

(Structural Engineering and Construction Management)

04 CE 6401- Analytical methods in Engineering

Max. Marks : 60

Duration: 3 Hours

Part-A

Answer all questions

Each question carries 3marks

1. Solve the differential equation $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + \frac{dy}{dx} + 6y = 0$
2. Solve $\frac{\partial^2z}{\partial x^2} - \frac{\partial^2z}{\partial x\partial y} - 6\frac{\partial^2z}{\partial y^2} = 0$
3. Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x, 0) = 6e^{-3x}$
4. Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x, 0) = 6e^{-3x}$
5. Classify the partial differential equation $\frac{\partial^2z}{\partial x^2} = \frac{\partial^2z}{\partial y^2}$
6. In which part of the xy -plane the following equation is elliptic

$$\frac{\partial^2u}{\partial x^2} + \frac{\partial^2u}{\partial x\partial y} + (x^2 + 4y^2)\frac{\partial^2u}{\partial y^2} = 2\sin xy.$$

7. Derive standard 5-point formula
8. Derive diagonal 5-point formula

(8 x 3 = 24 marks)

Part-B

Answer one choice in each question

Each question carries 6marks

9.

a. Solve $(3x + 2)^2 \frac{d^2y}{dx^2} + 3(3x + 2) \frac{dy}{dx} - 36y = 3x^2 + 4x + 1$

Or

b. Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2} + 4y = 4\sec^2 2x$

10.

a. Solve $(y - z)p + (x - y)q = z - x$

Or

b. Show that the equations $\frac{\partial z}{\partial x} = (x + y)^2$, $\frac{\partial z}{\partial y} = x^2 + 2xy + y^2$ are compatible and solve them.

11.

a. Solve $z^2 = pqxy$

Or

b. Solve $r - 4s + 4t = e^{2x+y}$

12.

a. A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = a \sin \frac{\pi x}{l}$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by $y(x, t) = a \sin \frac{\pi x}{l} \cos \frac{\pi ct}{l}$

Or

b. An infinitely long plane uniform plate is bounded by two parallel edges and an end at right angles to them. The breadth is π ; this end is maintained at a temperature u_0 at all points and other edges are at zero temperature. Determine the temperature at any point of the plate in steady state.

13.

a. Derive the finite difference approximation for the partial derivative u_{xx} with diagram.

Or

b. Derive the finite difference approximation for the partial derivative u_{yy} with diagram.

14.

a. Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the square mesh of the following fig. with boundary values

$$\begin{aligned} u(1,0) &= 500, u(2,0) = 1000, u(3,0) = 500, u(4,0) = 0, u(0,0) = 0, \\ u(0,1) &= 1000, u(0,2) = 2000, u(0,3) = 1000, u(0,4) = 1000, \\ u(1,4) &= 500, u(2,4) = 1000, u(3,4) = 500, u(4,4) = 0, u(4,1) = 1000 \\ u(4,2) &= 2000, u(4,3) = 1000 \end{aligned}$$

| | | | |
|---|---------|-------|---------|
| | u_1 C | u_2 | u_3 |
| A | u_4 | u_5 | u_6 B |
| | u_7 | u_8 | u_9 |
| | D | | |

Or

b. Evaluate the pivotal values of the equation $u_{tt} = 16u_{xx}$, taking $h=1$ upto $t = 1.25$. The boundary conditions are $u(0, t) = u(5, t) = 0$, $u_i(x, 0) = 0$ and $u(x, 0) = x^2(5 - x)$.

(6 x 6 = 36 marks)