A **530B2** **Total pages: 2**

Register No: …… Name: .....

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FIRST SEMESTER B.TECH DEGREE EXAMINATION(R), NOVEMBER 2024

Common for All Programmes

(2024 SCHEME)

Course Code : 24BST1101

Course Name : Linear Algebra and Calculus

Max. Marks : 60 **Duration:2.5 Hours**

Use of non programmable calculators can be permitted.

PART A

(Answer all questions. Each question carries 3 marks)

1. Reduce to Echelon form and hence find the rank of the matrix $A = \begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \\ 21 & -21 & 0 & -15 \end{bmatrix}$.

2. Find the sum and product of the eigenvalues of $\begin{bmatrix} 3 & 1 & -1 \\ 0 & 2 & 6 \\ 0 & 0 & 6 \end{bmatrix}$.

3. Given $f(x, y) = e^x \sin y$. Check whether f satisfies the Laplace equation $f_{xx} + f_{yy} = 0$.

4. Evaluate the integral,
   $\int_{-1}^{2} \int_{0}^{2} \int_{0}^{1} (x^2 + y^2 + z^2)\, dxdydz$

5. Find the Taylor series for $\frac{1}{x}$ about $x = 1$.

PART B

(Answer one full question from each module, each question carries 9 marks)

MODULE I

6. Check the consistency and solve the following system $\begin{matrix} x + y + z = 1 \\ x + 2y + 4z = 2 \\ x + 4y + 10z = 4 \end{matrix}$. 9

OR

7. Solve the linear system by Gauss elimination method given the augmented matrix 9
   $\left[\begin{array}{cccc:c} 1 & 0 & 0 & 4 & -1 \\ 0 & 1 & 0 & 2 & 6 \\ 0 & 3 & 1 & 3 & 2 \end{array}\right]$

MODULE II

8. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. 9

OR

9. Find out what type of conic section the following forms represents and transform it to principal axes $3x_1^2 + 22x_1x_2 + 3x_2^2 = 0$. 9

MODULE III

10. Find the local linear approximation $L(x, y)$ to $f(x, y) = \sqrt{x^2 + y^2}$ at the point (3,4). Compare the error in approximating f by L at the specified point $Q(3.04, 3.98)$ 9

OR

11. Suppose that $w = x^2 + y^2 - z^2$ and 9
$x = \rho \sin\phi \cos\theta, \;\; y = \rho \sin\phi \sin\theta, \;\; z = \rho \cos\phi.$
Use appropriate forms of the chain rule to find $\frac{\partial w}{\partial \rho}$, $\frac{\partial w}{\partial \phi}$ and $\frac{\partial w}{\partial \theta}$.

MODULE IV

12. Use cylindrical co-ordinates to evaluate 9
$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{0}^{9-x^2-y^2} x^2 \, dzdydx$

OR

13. Evaluate 9
$\iint (x^2 + y^2) dxdy$
over the region in the positive quadrant for which $x + y \leq 1$

MODULE V

14. Obtain the half range Fourier sine series for e^x in $0 < x < L$. 9

OR

15. Find the Fourier series for the function $f(x) = e^{-x}$ defined in $(0, 2\pi)$. 9

****