120A1

A Total Pages: **3**

Register No.: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING
KOTTAYAM, KERALA

(AN AUTONOMOUS COLLEGE AFFILIATED TO
APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FIRST SEMESTER B.TECH DEGREE EXAMINATION (S,FE), NOVEMBER 2024

Course Code: **20MAT101**

Course Name: **LINEAR ALGEBRA AND CALCULUS**

Max. Marks: **100** **Duration:** **3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}$.

2. Find all eigenvalues of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

3. Find $\frac{\partial^2 f}{\partial x \partial y}$ at (1,1) where $f = x^2 y$.

4. Show that the function $f(x, y) = x^2 - y^2 + 2xy$ satisfies Laplace equation $f_{xx} + f_{yy} = 0$.

5. Evaluate $\int_1^3 \int_2^4 (40 - 2xy) dydx$.

6. Use double integral, find the volume of the solid that is bounded above by the plane $z = 4 - x - y$ and below the rectangle $R = [0,1] \times [0,2]$.

7. Determine whether the series $\sum_{k=0}^{\infty} \frac{5}{4^k}$ converges or not, and if so find its sum.

8. Use appropriate test, determine whether the series $\sum_{k=1}^{\infty} \frac{1}{2k^2+k}$ converges or diverges.

9. Find the Maclaurin series expansion of $\cos x$

10. If $f(x)$ is a function defined in the interval $[-l, l]$, then write Euler's formula to find a_0, a_n & b_n in the Fourier series expansion of $f(x)$

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Solve the following simultaneous equation (6)

$$x + 3y - 2z = 0$$
$$2x - y + 4z = 0$$
$$x - 11y + 14z = 0$$

b) Check whether the matrix $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$ is diagonalizable. If yes, find P such that $D = P^{-1}AP$ is diagonal matrix. (8)

OR

12. a) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. (8)

b) Investigate for what values of a, b the simultaneous equations; (6)

$$x + y + 2z = 2 \ ; \quad 2x - y + 3z = 2 \ ; \quad 5x - y + az = b$$

Have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

MODULE II

13. a) Let $f(x, y, z) = x^3y^5z^7 + xy^2 + y^3z$. Find (8)

(i) f_{xy} (ii) f_{yz} (iii) f_{xz} (iv) f_{xx}

b) Find the local linear approximation L of $f(x, y,) = \sqrt{x^2 + y^2}$ at $P = (3,4)$. Also compare the error in approximating f by L at $Q = (3.04,3.98)$ with the distance between P and Q. (6)

OR

14. a) Let f be a differentiable function of one variable an let $w = f(u)$, where $u = x + 2y + 3z$. Show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 6\frac{\partial w}{\partial u}$ (8)

b) If $z = log\sqrt{x^2 + y^2}$, prove that (i) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 1$ (ii) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$. (6)

MODULE III

15. a) Use polar co-ordinates to evaluate $\iint_R e^{-(x^2+y^2)}\, dA$ when $R$is the region enclosed by the circle, $x^2 + y^2 = 1$ (7)

b) Use a double integral to find the area of the region R enclosed between the parabolas $y = \frac{1}{2}x^2$ and the line $y = 2x$. (7)

OR

16. a) Evaluate $\int_0^\infty \int_0^x xe^{-\frac{x^2}{y}}\, dydx$ by changing the order of integration. (7)

b) Use a triple integral to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$. (7)

MODULE IV

17. a) Test the convergence of the following series (8)

i) $\sum_{k=1}^{\infty} \frac{1}{k!}$ ii) $\sum_{k=1}^{\infty} \frac{k}{2^k}$

b) Test the convergence of the following series (6)

i) $\sum_{k=2}^{\infty} \left(\frac{4k-5}{2k+1}\right)^k$ ii) $\sum_{k=1}^{\infty} \frac{1}{(\ln(k+1))^k}$

OR

18. a) Show that the series $\sum_{1}^{\infty} \frac{[(k+1)r]^k}{k^{k+1}}$ is convergent if $r < 1$ and divergent if $r \geq 1$. (8)

b) Check whether the series $\sum_{k=1}^{\infty} (-1)^k \frac{(2k)!}{k^{2k}}$ converges absolutely. (6)

MODULE V

19. a) Let $f(x) = \begin{cases} -k, & -\pi \leq x \leq 0 \\ k, & 0 \leq x \leq \pi \end{cases}$. Find the Fourier series expansion of $f(x)$. (8)

b) Write the Taylor series expansion for the function $f(x) = \frac{1}{x}$ about $x = 1$ (6)

OR

20. a) Express $f(x) = x$ as a half range sine series in $0 < x < 2$ (8)

b) Find the Fourier series expansion of $f(x) = x^2$ in $(-\pi, \pi)$. (6)

****