H **931A2** Total Pages: **5**

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SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FOURTH SEMESTER B.TECH DEGREE EXAMINATION (R), MAY 2024

(2020 SCHEME)

Course Code : **20CST294**

Course Name: **Computational Fundamentals for Machine Learning**

Max. Marks : **100** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Solve the linear system:

$$x-y+2z=5$$
$$2x-2y+4z=10$$
$$3x-3y+6z=15$$

2. Find the coordinate vector of w relative to the basis S=u_1,u_2 for R_2 given u_1=(2,-4),u_2=(3,8) and w=(1,1)
3. Let $u = (2,-1, 3)$ and $a = (4,-1, 2)$; Find the vector component of u along a and the vector component of u orthogonal to a.
4. The planes: $x + 2y - 2z = 3$ and $2x + 4y - 4z = 7$ are parallel since their normals, $(1, 2,-2)$ and $(2, 4,-4)$, are parallel vectors. Find the distance between these planes.
5. Compute $f'(x)$ of the logistic sigmoid $f(x)= \frac{1}{1+e^{-x}}$
6. Find the gradient of $f(x,y)= x^2y$ at the point (3,2)
7. Define the following i) conditional probability ii) mutually exclusive events
8. Calculate the probability of normal distribution with the population mean 2, standard deviation 3 or random variable 5
9. Show that every affine function $f(x)=ax+b$, $x \in R$ is convex.
10. Is the function $f(x,y)=2x^2 + y^2 + 6xy - x + 3y - 7$ convex, concave, or neither? Justify your answer.

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Solve the system of linear equations using Gauss-Jordan Elimination (8)

$$x + 2y + 3z = 5$$
$$2x + 5y + 3z = 3$$
$$x + 8z = 17$$

b) Show that the vectors $v_1 = (1, 2, 1)$, $v_2 = (2, 9, 0)$, *and* $v_3 = (3, 3, 4)$ form a basis for R_3. (6)

OR

12. a) Consider the vectors u=(1,2,-1) and v=(6,4,2) in R_3. Show that w=(9,2,7) is a linear combination of u and v, and w'=(4,-1,8) is *not* a linear combination of u and v. (7)

b) Find the rank and nullity of the matrix

$$A=\begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$$ (7)

MODULE II

13. a) Find the matrix P that diagonalizes a matrix A

$$A=\begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$ (7)

b) Show that the vectors w_1=(0,2,0) w_2=(3,0,3) w_3=(-4,0,4) for an orthogonal basis for R_3 with Euclidean inner product, and use that basis to find an orthonormal basis by normalizing each vector. Express the vector u=(1,2,4) as a linear combination of orthonormal basis vectors obtained. (7)

OR

14. a) What do you meant by singular value decomposition? Find a singular value decomposition of the matrix

$$A=\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$ (8)

b) Assume that the vector space has the Euclidean inner product. Apply the Gram–Schmidt process to transform the basis vectors u_1=(1,1,1) u_2=(0,1,1) u_3=(0,0,1) into an orthogonal basis(v_1,v_2,v_3), and then normalize the orthogonal basis vectors to obtain an orthonormal basis(q_1,q_2,q_3) . (6)

MODULE III

15. a) Find the direction of greatest increase of the function $f(x,y)=4x^2+y^2+2y$ at the point $P(1,2)$ (6)

b) A skier is on a mountain with equation

$$z = 100 - 0.4x^2 - 0.3y^2$$

where z denotes height. The skier is located at the point with xy-coordinates (1,1) and wants to ski downhill along the steepest possible path. In which direction (indicated by a vector (a;b) in the xy-plane) should the skier begin skiing? (8)

OR

16. a) Prove that Expectation of sum of two r.v.'s: $\mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y]$ (6)

b) Suppose we have the following joint density function (8)

$$f(x, y) = c(x + y), 0 \leq x \leq 1, 0 \leq y \leq 2$$

(i) Find the value of c ?
(ii) Find the marginal distributions of X and Y?

MODULE IV

17. a) A doctor is called to see a sick child. The doctor has prior information that 90% of sick children in that neighborhood have the flu, while the other 10% are sick with measles. Let F stand for an event of a child being sick with flu and M stand for an event of a child being sick with measles. Assume for simplicity that F ∪ M = Ω, i.e., that there no other maladies in that neighborhood. A well-known symptom of measles is a rash (the event of having which we denote R). Assume that the probability of having a rash if one has measles is P(R | M) = 0.95. However, occasionally children with flu also develop rash, and the probability of having a rash if one has flu is P(R | F) = 0.08. Upon examining the child, the doctor finds a rash. What is the probability that the child has measles? (6)

b) Suppose that you are watching a video clip consisting of thirty frames per second from YouTube. Each frame consists of 1000 packets. When a frame is about to be played out, if five or more packets of the frame are lost (i.e., if you have received 995 or fewer packets of the frame), you will experience buffering during that frame. Each packet is lost with the probability θ, independently of other packets. Suppose θ = 0.001. Using the Poisson approximation, find the probability that you experience buffering during a given frame (8)

OR

18. a) The number of ships to arrive at a harbor on any given day is a (6)

random variable represented by x. The probability distribution for x is:

x	10	11	12	13	14
P(x)	.4	.2	.2	.1	.1

Find the probability that on a given day:

a. exactly 14 ships arrive

b. At least 12 ships arrive

c. At most 11 ships arrive

b) A random square has a side length that is a uniform [0, 1] random variable. Find the expected area of the square. (8)

MODULE V

19. a) Determine the maximum and minimum values of the function $f(x)=12x^5 - 45x^4 + 40x^3 + 5$ (7)

b) Use the method of Lagrangian Multipliers to find the point that represent the minimum value for $f(x,y)=x^2 + y^2$ subject to the constraint $x + 4y = 20$ (7)

OR

20. a) Find the extreme points of the function $f(x_1,x_2)=x_1{}^3 + x_2{}^3 + 2x_1{}^2 + 4x_2{}^2 + 6$ (7)

b) A furniture company produces inexpensive tables and chairs. The production process for each is similar in that both require a certain number of hours of carpentry work and a certain number of labour hours in the painting department. Each table takes 4 hours of carpentry and 2 hours in the painting department. Each chair requires 3 hours of carpentry and 1 hour in the painting department. During the current production period, 240 hours of carpentry time are available and 100 hours in painting is available. Each table sold yields a profit of 7; each chair produced is sold for a profit of 5.. Find the best combination of tables and chairs to manufacture in order to reach the maximum profit. (7)

Department	**Hours Required to make 1 Unit**		**Available Hours**
	Tables	**Chairs**	
Carpentry	4	3	240
Painting	2	1	100
Profit	7	5	

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