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# SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

**FOURTH SEMESTER B.TECH DEGREE EXAMINATION (R), MAY 2024**

**(2020 SCHEME)**

**Course Code :** **20CST284**

**Course Name:** **Mathematics for Machine Learning**

**Max. Marks :** **100** **Duration: 3 Hours**

## PART A

***(Answer all questions. Each question carries 3 marks)***

1. Check whether the vectors $v_1$, $v_2$, $v_3$ are linearly independent

   $v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ $v_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ $v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

2. Find the basis and dimension of vector space spanned by vectors

   $v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ $v_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ $v_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

3. What is $L_1$ and $L_2$ norm of a vector? The vector has an initial point at $P = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$ and an endpoint at $Q = \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix}$. What is the vector's length?

4. Find the angle between $a = \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 7 \\ -3 \end{bmatrix}$.

5. Let w=f(x-y,y-z,z-x) .Show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$

6. Consider $f(x, y) = 2\sqrt{x} + x^2 \ln(y)$
   (i) Find the gradient of f
   (ii) Evaluate the gradient at (4,1)

7. If two cards are drawn simultaneously from a pack of cards, what is the probability that both will be jacks or both are queens?

8. Define (i) conditional probability (ii) Bayes Theorem

9. Compare gradient descent and stochastic gradient descent methods. Explain with example when one method is preferred over other

10. What is optimization? What is constrained optimization and unconstrained optimization?

## PART B

***(Answer one full question from each module, each question carries 14 marks)***

## MODULE I

11. a) For what values of 'n' and 'm' the system of linear equations (14)
x+y+z=6
x+2y+5z=10
2x+3y+nz=m
have (i) no solution (ii) unique solution (iii) more than one solution (iv) find the solution for n=2 and m=8.

**OR**

12. a) Consider $A=\left\{\begin{bmatrix}1\\0\\1\end{bmatrix},\begin{bmatrix}1\\1\\0\end{bmatrix},\begin{bmatrix}2\\2\\-3\end{bmatrix}\right\}$ and $B=\left\{\begin{bmatrix}3\\1\\1\end{bmatrix},\begin{bmatrix}0\\0\\1\end{bmatrix},\begin{bmatrix}-1\\2\\0\end{bmatrix}\right\}$. (14)
Both A and B are bases for $\mathbb{R}^3$
(i) Find $P_{B\to A}$
(ii) Find $P_{A\to B}$
(iii)Let $X=2a_1 - a_2 + a_3$ , $[X]_A = \begin{bmatrix}2\\-1\\3\end{bmatrix}$ Find $[X]_B$

## MODULE II

13. a) A is a 2 × 2 matrix with eigenvalues $\lambda_1 = 2$ and $\lambda_2 = 5$, and corresponding eigenvectors $v_1 = [1, 1]$ and $v_2 = [-1, 1]$. (10)
(i) Find the eigen decomposition of A and find matrix A.
(ii) Compute $A^{10}$ using the eigen decomposition.

b) Let Π be the plane in $R^3$ spanned by vectors $x_1 = (1, 2, 2)$ and $x_2 = (-1, 0, 2)$. Find an orthonormal basis for Π. (4)

**OR**

14. a) Let V be the plane spanned by vectors, $v_1 = (1, 1, 0)$ and $v_2 = (0, 1, 1)$. Find $V^{\perp}$. (4)

b) Find the orthogonal projection matrix onto the plane x+y-z=0. (10)

## MODULE III

15. a) Write the definition of derivative. (8)
Derive the result derivation of $x^n$ using the definition of derivative.

b) Calculate Jacobians for the following mappings (6)
(i)w=x-2y , z=2x+y
(ii) w=2x-3y, z=5x+7y

**OR**

16. a) Use the chain rule to find derivatives of the following, where (6)
$f(x, y, z) = x^2 + yz$,
$g(x, y) = y^3 + xy$,

h(x) = sin x

b) Find the Taylor Series expansion of $f(x, y) = x^2 + 2xy + y^3$ at the point (1, 2). (8)

**MODULE IV**

17. a) The weight of apples produced by a farm follows a normal distribution with a mean of 150 grams and a standard deviation of 20 grams.

(i) What is the probability that a randomly selected apple from the farm weighs between 130 grams and 170 grams? (6)

(ii) What weight does an apple need to be in order to be in the top 10% by weight produced on the farm?

(iii) If a random sample of 50 apples is taken from the farm, what is the probability that the sample mean weight is between 145 grams and 155 grams?

b) A store has three different types of computers for sale: brand A, brand B, and brand C. The proportion of computers sold in the store is as follows:

40% of computers sold are brand A
30% of computers sold are brand B
30% of computers sold are brand C

Suppose that 5% of brand A computers are defective, 10% of brand B computers are defective, and 15% of brand C computers are defective. (8)

(i) What is the probability that a randomly selected computer from the store is defective?

(ii) If a computer is defective, what is the probability that it is a brand A computer?

(iii) If a computer is not defective, what is the probability that it is a brand B computer?

**OR**

18. a) Suppose that a factory produces bolts, and 95% of the bolts meet the required specifications, while 5% are defective. A sample of 10 bolts is selected at random from the factory's production line.

(i) What is the probability that all 10 bolts in the sample meet the required specifications? (6)

(ii) What is the probability that at least one bolt in the sample is defective?

(iii) Suppose that two bolts are randomly selected from the sample without replacement. What is the probability that both bolts meet the required specifications?

b) A medical test for a certain disease is known to have a false positive rate of 2% and a false negative rate of 5%. The prevalence of the disease in the population is 1 in 5000. Suppose a person tests positive for the disease. (8)

(i) What is the probability that the person actually has the disease?
(ii) If the person tests negative for the disease, what is the probability that the person actually does not have the disease?

**MODULE V**

19. a) Consider the function $f(x) = x^4 - 3x^3 + 2$.Use gradient descent to find the minimum of this function.

(a) Calculate the gradient of f(x).
(b) Derive the update rule for gradient descent. (8)
(c) Suppose we start with an initial guess of $x_0 = 1$. Use gradient descent with a step size of 0.1 to find the minimum of f(x) to within an error tolerance of 0.01. Show your calculations and report the final estimate of the minimum after 10 iterations.

b) Find the local minimizers and maximizers for the following functions if they exist:
(a) $f(x) = x^2 + \cos x$ (6)
(b) $f(x, y) = x^2 - 4x + 2y^2 + 7$
(c) $f(x, y, z) = (2x - y)^2 + (y - z)^2 + (z - 1)^2$

**OR**

20. a) Maximize z = 4x + 3y subject to the following constraints:
$2x + y \leq 6$
$x + 2y \leq 8$ (8)
$x \geq 0$
$y \geq 0$

b) Consider the function $f(x, y) = x^3 + 3xy^2$
(i) Find all the critical points of f.
(ii) Calculate the Hessian matrix of f. (6)
(iii) Determine whether each critical point is a local maximum, local minimum, or saddle point by examining the eigenvalues of the Hessian matrix.

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