Register No.: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

SECOND SEMESTER B.TECH DEGREE EXAMINATION (R,S), MAY 2024

(2020 SCHEME)

Course Code : **20MAT102**

Course Name: **Vector Calculus, Differential Equations and Transforms**

Max. Marks : **100** **Duration: 3 Hours**

Non- programmable calculator may be permitted

PART A

(Answer all questions. Each question carries 3 marks)

1. Find the displacement and the distance travelled over the time interval $0 \le t \le \pi$, where the position vector is $\vec{r}(t) = (1 - 3sint)\hat{\imath} + 3cost\hat{\jmath}$.
2. Find a unit vector in the direction in which f increases most rapidly at P and find the rate of change of f at P in that direction $f(x, y) = 4x^3y$; $P(-1,1)$.
3. Determine whether the vector field $\vec{F}(x, y, z) = xy\hat{\imath} - 2xy\hat{\jmath} + y^2\hat{k}$ is free of sources and sinks. If it is not, locate them.
4. Using Green's theorem find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
5. Check whether e^{4x}, $e^{-1.5x}$ are linearly dependent or independent using Wronskian.
6. Find a general solution of $y'' + 8y' + 15y = 0$
7. Find the Laplace transform of $sin^2\, t$
8. Find the inverse Laplace transform of $cot^{-1}\left(\frac{s}{a}\right)$
9. Find the Fourier sine Transform of $f(x) = e^{-ax}$
10. Find Fourier cosine integral representing of $f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & x > 1 \end{cases}$

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11\. a) Find the directional derivative of $f(x, y, z) = x^2y - yz^3 + z$ at $P(1, -2, 0)$ in the direction of the vector $\vec{a} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$. (7)

b) Determine whether $\vec{F} = e^y\hat{\imath} + xe^y\hat{\jmath}$ is conservative vector field. If so, find the scalar potential. Hence evaluate the work done in moving an object in this field from (1,0) to (−1,0) along the upper semicircular path of the circle $x^2 + y^2 = 1$. (7)

OR

12. a) Find the divergence and the curl of the vector field $\vec{F}(x,y,z) = e^{xy}\hat{\imath} - 2\cos y\hat{\jmath} + \sin^2 z\hat{k}$. (7)

b) Evaluate $\int_c yzdx - xzdy + xydz$, where C is the path $x = e^t$, $y = e^{2t}, z = e^{-t}$ where $0 \le t \le 1$. (7)

MODULE II

13. a) Use Green's theorem to evaluate $\oint_c (e^x + y^2)\,dx + (e^y + x^2)dy$ where C is the boundary of the region between $y = x^2$ and $y = 2x$. (7)

b) Find the outward flux of the vector field $\vec{F} = x^3\hat{\imath} + y^3\hat{\jmath} + z^3\hat{k}$ across the surface of the region that is enclosed by the circular cylinder $x^2 + y^2 = 4$ and the plane z= 0 and z= 4. (7)

OR

14. a) Evaluate work done by $\vec{F} = (e^{2x} - y^3)\hat{\imath} + (\sin y + x^3)\hat{\jmath}$ on a particle that travels once around a circle $x^2 + y^2 = 4$ in counterclockwise direction, using Green's theorem. (7)

b) Apply Stokes theorem to evaluate the integral $\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F} = xy\hat{\imath} + yz\hat{\jmath} + zx\hat{k}$ and C is the triangle in the plane $x + y + z = 1$ with vertices $(1,0,0), (0,1,0)$ *and* $(0,0,1)$ with a counterclockwise orientation looking from the first octant towards the origin. (7)

MODULE III

15. a) Using the method of variation of parameter solve the differential equation $(D^2 - 4D + 5)y = e^{2x}\operatorname{cosec} x$. (7)

b) Solve the initial value problem $x^2y'' - 3xy' + 4y = 0$; $y(1) = \pi\,, y'(1) = 4\pi$. (7)

OR

16. a) Solve $(D^2 - 4D + 3I)y = e^x$. (7)

b) Solve the initial value problem $y'' + y' + 0.25y = 0$, $y(0) = 3\,, y'(0) = -3.5$. (7)

MODULE IV

17. a) Using Laplace transform, solve the differential equation $y'' - 3y' + 2y = 4e^{2t}, y(0) = -3,\ y'(0) = 5$. (7)

b) Find (i) $L(e^{3t}\sinh t)$ (ii) $L^{-1}\left(\frac{2}{s^4} - \frac{48}{s^6}\right)$ (7)

OR

18. a) Apply convolution theorem to find the inverse Laplace transform of $\frac{16}{(s-2)(s+2)^2}$. (7)

b) Find the Laplace transform of (i) $\int_0^t \cos t\,dt$ (ii) $te^{2t}\sin 3t$ (7)

MODULE V

19. a) Find the Fourier integral representation of the $f(x) = \begin{cases} 2 & |x| < 1 \\ 0 & |x| > 1 \end{cases}$ (7)

b) Find the Fourier sine transform of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 0 \end{cases}$ (7)

OR

20. a) Find the Fourier cosine transform of e^{-x^2}. (7)

b) Find the inverse Fourier transform of $F(w) = \begin{cases} 1 \ if \ |w| < w_0 \\ 0 \ if \ |w| > w_0 \end{cases}$ (7)

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