Register No.: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

THIRD SEMESTER B.TECH DEGREE EXAMINATION (R,S), DECEMBER 2023
COMMON TO CE,CH,EC,EE,FT,ME,RA
(2020 SCHEME)

Course Code : **20MAT201**

Course Name: **Partial Differential Equations and Complex Analysis**

Max. Marks : **100** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Determine the partial differential equation by eliminating arbitrary constants from $z = (x^2 + a^2)(y^2 + b^2)$.
2. Solve $x^2 p + y^2 q = z^2$.
3. List any three assumptions in deriving one dimensional heat equation.
4. Write the boundary conditions and initial conditions of the string of length l which is initially at rest in equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = \mu_0 \sin\left(\frac{\pi x}{l}\right), 0 < x < l$
5. Determine Ref and Imf of $f(z) = e^{iz}$.
6. Determine the critical points of the function $w = \frac{1}{2}(z + \frac{1}{z})$.
7. Evaluate $\int \bar{z}\, dz$ over the circle $|z| = 1$.
8. Find the Taylor series expansion of $f(z) = \frac{1}{1+z}$ about $z = 3$.
9. Determine the singularities of the function $f(z) = \frac{ze^z}{z^2+4}$.
10. Find the residue of the function $\frac{1-e^{2z}}{z^4}$ at $z = 0$.

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Solve $(y - z)p + (x - y)q = (z - x)$. (7)

    b) Using the method of separation of variables solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x, 0) = 3e^{-5x}$. (7)

OR

12. a) Solve $\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial t} - u = 0,\ u(x, 0) = 6e^{-3x}$ using method of separation of variables. (7)

b) Find the partial differential equation of all spheres with center on z-axis and radius a. (7)

MODULE II

13. a) Derive the solution of one-dimensional wave equation. (7)

b) A rod of length L is heated in such a way that its ends A and B are at zero temperature. If initially its temperature is given by $u = \frac{cx(L-x)}{L^2}$, $0 \leq x \leq L$, find the temperature at time t. (7)

OR

14. a) A uniform elastic string of length 60cms is fastened at two ends. The initial displacement is given by $y(x,0) = 60x - x^2$ for $0 < x < 60$, while the initial velocity is zero. Evaluate the displacement function $y(x,t)$. (7)

b) Solve the one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(x,t) \neq \infty \; if \; t \to \infty; u(0,t) = 0 = u(\pi,t)$; $u(x,0) = \pi x - x^2, \quad 0 \leq x \leq \pi$. (7)

MODULE III

15. a) If f(z) is analytic in a domain D and |f(z)| is constant in D then show that f(z) is a constant. (7)

b) Examine whether $u = y^3 - 3x^2y$ is harmonic. Hence find its harmonic conjugate. (7)

OR

16. a) Show that $f(z) = \sin z$ is analytic everywhere. Also find $f'(z)$. (7)

b) Determine the region of the w-plane into which the triangular region bounded by $x = 1, y = 1$ and $x + y = 1$ is mapped by $w = z^2$ (7)

MODULE IV

17. a) Evaluate $\int_0^{1+i} (x^2 - ixy)dz$ along the path $y = x^2$. (7)

b) Evaluate $\oint secz \, dz$ over the circle $|z| = 1$. (7)

OR

18. a) Evaluate $\int_C \log z \, dz$ where C is the circle $|z|=1$. (7)

b) Evaluate $\int \frac{z^2+2z+3}{z^2-1} dz$ over $|z-1| = 1$. (7)

MODULE V

19. a) Determine the Laurent's series expansion of $f(z) = \frac{z}{(z+1)(z+2)}$ about $z = -2$ in $0 < |z+2| < 1$. (7)

b) Describe different types of singular points of an analytic function with examples. (7)

OR

20. a) Evaluate $\int \frac{\cos\pi z^2}{(z-1)(z-2)} dz$ over $|z| = 3$. (7)

b) Using contour integration, evaluate $\int_0^\infty \frac{1}{(x^2+a^2)^2}\, dx$. (7)

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