Register No.: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FIRST SEMESTER M.TECH DEGREE EXAMINATION (Regular), DECEMBER 2023

(2021 Scheme)

Course Code: **21GS101**

Course Name: **Applied Mathematics for Civil Engineers**

Max. Marks: **60** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Obtain the reduction formula for gamma function.
2. Find the Laplace transform of $e^{4t}\sin 2t\cos t$.
3. What is the difference between Fredholm and Volterra integral equation.
4. Find the partial differential equation of $z = ax + by + \sqrt{a^2 + b^2}$ by eliminating the arbitrary constants a and b.
5. Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u\,, where\; u(x,0) = 6e^{-3x}$
6. Express the transverse vibration of a solid object which is described by 3-Dimensional wave equation.
7. Determine whether the following equation is elliptic or hyperbolic $$(1+x^2)\frac{\partial^2 u}{\partial x^2} + (5+2x^2)\frac{\partial^2 u}{\partial x\partial t} + (4+x^2)\frac{\partial^2 u}{\partial t^2} = 0$$
8. In which parts of the (X,Y) plane is the following equation elliptic $u_{xx} + u_{yx} + (x^2 + 44y^2)u_{yy} = 2\sin xy$

PART B

(Answer one full question from each module, each question carries 6 marks)

MODULE I

9. Obtain the generating function for $J_{n(x)}$ (6)

OR

10. Solve $(1-x^2)\frac{d^2y}{dx^2} - x\,\frac{dy}{dx} + 4y = 0$ (6)

MODULE II

11. Solve by the method of Laplace transforms the equation $y''' + 2y'' - y' - 2y = 0$ given $y(0) = 0\,, y''(0) = 6$. (6)

OR

12. Find the Fourier sine transform of $e^{-|x|}$, hence show that $\int_0^\infty \frac{x \sin mx}{1+x^2}\,dx = \frac{\pi e^{-m}}{2}\,, m > 0$ (6)

MODULE III

13. Solve the Abel's integral equation $\int_0^x \frac{y(t)}{\sqrt{x-t}}\,dt = 1 + 2x - x^2$ (6)

OR

14. Using the method of successive approximations, solve the Volterra integral equation $y(x) = 1 + x + \int_0^x (x-1)y(t)dt$. (6)

MODULE IV

15. Find the equation of integral surface of differential equation $2y(z-3)p + (2x-z)q = y(2x-3)$ which passes through circle $z = 0, x^2 + y^2 = 2x$ (6)

OR

16. Solve $2xz - px^2 - 2qxy + pq = 0$ (6)

MODULE V

17. Solve by the method of separation of variables $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ (6)

OR

18. A tightly stretched string with fixed end points $x = 0 and\ x = l$ is initially in a position given by $y = y_0 sin^3(\frac{\pi x}{l})$. If it is released from rest from this position, find the displacement function $y(x,t)$. (6)

MODULE VI

19. Solve the boundary value problem $u_{tt} = u_{xx}$ with the conditions u(0,t)=u(1,t)=0,u(x,0)=$\frac{1}{2}x(1-x) and\ u_t(x,0) = 0, taking\ h = k = 0.1\ for$ $0 \le t \le 4$. Compare your solution with the exact solution at x=0.5 and t=0.3. (6)

OR

(6)

20. Solve $\nabla^2 u = 0$ under the conditions (h=1,k=1) u(0,y)=0, u(4,y)=12+y for 0≤y≤4; u(x,0)=3x, u(x,4)=x^2 for 0≤x≤4.

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