Register No.: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

SECOND SEMESTER B.TECH DEGREE EXAMINATION (R), MAY 2023

(2020 SCHEME)

Course Code : **20MAT102**

Course Name: **Vector Calculus, Differential Equations and Transforms**

Max. Marks : **100** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Find the velocity, speed and acceleration at the given time t of a particle moving along the curve $x = 1 + 3t\,, y = 3 - 4t\,, z = 7 + 3t$ at $t = 2$.
2. Find curl F at the point $(1, -1, 1)$ when $\mathrm{F} = \mathrm{xz}^3\,\mathbf{i} - 2\mathrm{x}^2\mathrm{yz}\,\mathbf{j} + 2\mathrm{yz}^4\,\mathbf{k}$.
3. Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, using Green's theorem.
4. Let $F(x, y, z) = a\,\boldsymbol{i} + b\,\boldsymbol{j} + c\,\boldsymbol{k}$ be a constant vector field and σ be the surface of a solid G. Use divergence theorem to show that flux of F across σ is zero.
5. Check whether the function $y = a\cos x + b\sin x$ is a solution of $y'' + y = 0$.
6. Solve $y'' + 5y' + 6y = 0$.
7. Derive the Laplace transform of e^{at} where a is a constant and $t \geq 0$.
8. Find the Laplace transform of $\sin^2 t$.
9. Write the Fourier integral representation of a function $f(x)$.
10. Find the Fourier cosine transformation of $f(x) = \begin{cases} K & if\ 0 < x < a \\ 0 & if\ x > a \end{cases}$

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

11. a) Find the work done by the force field $\boldsymbol{F} = xy\,\boldsymbol{i} + x^3\,\boldsymbol{j}$ on a particle that moves along the curve $C : x = y^2$ from (0,0) to (1,1). (7)

    b) Determine whether $\boldsymbol{F} = 2\,e^{2x}\cos 2y\,\boldsymbol{i} - 2e^{2x}\sin 2y\,\boldsymbol{j}$ is a conservative vector field. If so find a potential function for it. (7)

OR

12. a) Let $f(x,y) = x^2e^y$, find the maximum value of a directional derivative at $(-2,0)$ and find the unit vector in the direction in which the maximum value occurs. (7)

b) Evaluate $\int_C 3xy\ dy$, where C is the line segment joining (0,0) and (1,2) with the given orientation (7)

(i) Oriented from (0,0) to (1,2)

(ii) Oriented from (1,2) to (0,0).

MODULE II

13. a) Using Green's theorem evaluate the work done by $\boldsymbol{F} = (e^{2x} - y^3)\boldsymbol{i} + (\sin y + x^3)\,\boldsymbol{j}$ on a particle that travels around the circle $x^2 + y^2 = 4$ in counter clock wise direction. (7)

b) Define source and sink of a vector field **F**. Determine whether the vector field $\boldsymbol{F}(x,y,z) = x^3\boldsymbol{i} + y^3\boldsymbol{j} + 2z^3\boldsymbol{k}$ is free of source and sink. If not locate them. (7)

OR

14. a) Use Divergence theorem to find the flux of $F(x,y,z) = (x^2 + y)\boldsymbol{i} + (xy)\boldsymbol{j} - (2xz + y)\boldsymbol{k}$ across the surface σ with outward orientation, were σ is the surface of the tetrahedron in the first octant bounded by $x + y + z = 2$ and the coordinate planes. (7)

b) Use Stoke's theorem to evaluate $\int_C F \cdot dr$ where $F(x,y,z) = xy\,\boldsymbol{i} + yz\,\boldsymbol{j} + zx\,\boldsymbol{k}\,; C$ is the triangle in the plane $x + y + z = 1$ with vertices $(1,0,0), (0,1,0)\ and\ (0,0,1)$ with counterclockwise orientation looking from the first octant towards origin. (7)

MODULE III

15. a) Solve the initial value problem (7)

$(x^2D^2 - 4xD + 6)y = 0\,, y(1) = 0.4\,, y'(1) = 0.$

b) Solve $y'' + 2y' + y = e^{2x}$. (7)

OR

16. a) Solve $y'' + 3y' + 2y = 12x^2$. (7)

b) Using method of variation of parameters solve $y'' + y = \sec x$. (7)

MODULE IV

17. a) Find the inverse Laplace transform of $\frac{s+1}{s^2+2s}$. (7)

b) Using Laplace Transform solve the initial value problem (7)

$y' + 4y = t\,, y(0) = 1.$

OR

18. a) Find the Laplace transform of t^3e^{-3t}. (7)

b) Solve the initial value problem $y'' - y = t\,, y(0) = 1\,, y'(0) = 1$. (7)

MODULE V

19. a) Find the Fourier integral representation of the function (7)

$$f(x) = \begin{cases} 1 \; if \; |x| < 1 \\ 0 \; if \; |x| > 1 \end{cases}.$$

b) Find the Fourier sine transform of $e^{-|x|}$,Hence evaluate $\int_0^\infty \frac{w \sin wx}{1+w^2} dw$. (7)

OR

20. a) Find the Fourier cosine transform of $f(x) = \begin{cases} 1 \;, 0 < x < 1 \\ -1 \;, 1 < x < 2 \\ 0 \;, \; x > 2 \end{cases}$ (7)

b) Obtain the Fourier sine transform of $\frac{e^{-ax}}{x}$. (7)

****