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Register No.:

Name: ..

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM) FIRST SEMESTER B.TECH DEGREE EXAMINATION (S), FEBRUARY 2023

(2020 SCHEME)

Max. Marks :	100	Duration: 3 Hours
Course Name:	LINEAR ALGEBRA AND CALCULAS	
Course Code :	20MAT101	

Non-Programmable Calculator May be Permitted

PART A

(Answer all questions. Each question carries 3 marks)

- 1. Determine the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & 5 \end{bmatrix}$.
- 2. Find the eigenvalues of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.
- 3. If $f(x, y, z) = x^3 y^2 z^4 + 2xy + z$, find $f_x(x, y, z)$, $f_y(x, y, z)$ and $f_z(x, y, z)$.
- 4. Find the rate of change of $z = \sin(y^2 4x)$ with respect to y at the point (3,1).
- 5. Evaluate the double integral $\int_0^1 \int_{-x}^{x^2} xy^2 dx dy$.
- 6. Evaluate the integral $\int_0^1 \int_0^1 \int_0^1 e^{(x+y+z)} dx dy dz$.
- 7. Check whether the series $\sum_{k=1}^{\infty} \left(\frac{k}{100}\right)^k$ converges or diverges.
- 8. Determine whether the series $\sum_{k=0}^{\infty} \frac{5}{4^k}$ is convergent and if so, find its sum.
- 9. Find the Taylor series expansion of e^x about x = -1.
- 10. Obtain the Fourier coefficient a_0 of the function f(x) = |x| in $-\pi < x < \pi$.

PART B

(Answer one full question from each module, each question carries 14 marks) MODULE I

11. a) Test for consistency and solve the following system of equations (7)

$$2x - y + 3z = 8$$
$$-x + 2y + z = 4$$
$$3x + y - 4z = 0$$

736A3

(7)

- Find the eigenvalues and corresponding eigenvectors of the matrix b) (7)31 1. 5
 - 13 1 1

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OR

12. (7)For what values of λ and μ the given system of equations a)

> x + y + z = 6x + 2y + 3z = 10 $x + 2y + \lambda z = \mu$

has (a) no solution (b) a unique solution and (c) infinite number of solutions.

Show that the matrix $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ is diagonalizable. Hence, (7)b) find *P* such that $P^{-1}AP$ is a diagonal matrix.

MODULE II

13. Let f be differentiable function of 3 variables and suppose that a) (7)w = f(x - y, y - z, z - x). Show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$.

b) Locate all the relative extrema and saddle points of (7) $f(x, y) = x^2 + xy - 2y - 3x + 1.$

OR

14. a) Given that
$$f(x, y) = y^3 e^{-5x}$$
, find
(a) $f_{xyy}(0,1)$ (b) $f_{xxx}(0,1)$ (c) $f_{yyxx}(0,1)$

Given that $z = e^{xy}$, x = 2u + v, $y = \frac{u}{v}$. Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ using the chain b) (7) rule.

MODULE III

15. (7)a) Reverse the order of integration and hence evaluate $\int_0^1 \int_x^1 \frac{x}{x^2+y^2} dy dx$.

Find the area bounded by the x - axis, y = 2x and x+y = 1 using the b) (7)double integration.

OR

736A3

- Use a double integral to find the volume of the tetrahedron bounded (7) 16. a) by the co-ordinate planes and the plane z = 4 - 4x - 2y.
 - Find the mass of the lamina with density $\delta(x, y) = x + 2y$ is bounded b) (7)by the x- axis, the line x = 1 and the curve $y = \sqrt{x}$.

MODULE IV

- 17. Check whether the following series converges. a)
 - $\sum_{k=1}^{\infty} \frac{1}{2k^2 + k}$ i. $\sum_{k=1}^{\infty} \frac{\frac{k^k}{k!}}{k!}$ ii.
 - Discuss the convergence of the series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+3}{k(k+1)}$ b) (7)

OR

- 18. a) (7)Determine the rational number representing the decimal number 0.412412412...
 - Check whether the following series converges. b) (7)

i.
$$\sum_{k=1}^{\infty} \frac{1}{\left(ln(k+1)\right)^{k}}$$

ii.
$$\sum_{k=1}^{\infty} k \left(\frac{1}{3}\right)^{k}$$

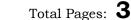
MODULE V

- 19. a) (7)Expand in a Fourier series, f(x) = x in the interval $0 < x < 2\pi$.
 - b) Find the Fourier cosine series expansion of $f(x) = x^2 - 2$ in (0,2). (7)

OR

Find the Fourier series to represent $x - x^2$ from $-\pi to \pi$ 20. a) (7)Obtain the half range sine series of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \end{cases}$ b) (7)

Page 3 of 3



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