# SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS) <br> (AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM) <br> FOURTH SEMESTER B.TECH DEGREE EXAMINATION (S), SEPT 2022 <br> COMMON TO CH, CE, FT, ME, RB <br> (2020 SCHEME) <br> Course Code : 20MAT202 <br> Course Name: Probability, Statistics and Numerical Methods <br> Max. Marks : 100 <br> Duration: 3 Hours 

(Non-programmable scientific calculators and statistical tables may be permitted)

## PART A <br> (Answer all questions. Each question carries 3 marks)

1. The probability distribution of a random variable is given by $P(X=x)=\frac{k}{2^{x}}$; where $x=0,1,2,3,4$. Find $k$.
2. Compute $\mathrm{E}(\mathrm{XY})$, the possible values of X are $\mathrm{x}=0,1,2,3$ and the possible values of Y are $\mathrm{y}=-1,1,2,3$

3 Find pdf of a random variable X given the distribution function

$$
F(x)=\left\{\begin{array}{cc}
1-\frac{1}{x^{2}} & ; x>1 \\
0 & ; x \leq 1
\end{array} . \text { Also find } P(X<3)\right.
$$

4. State Lindeberg-Levy Central Limit theorem.
5. A random sample of 20 observations produced a sample mean of $\bar{x}=92.4$ and $\mathrm{s}=25.8$. What is the value of the standard error of $\bar{x}$ ?
6. Define type I and type II error.
7. Find a positive solution of $\cos (x)+1=3 x$ using Newton- Raphson method correct to 4 decimal places.
8. What is the difference between Newton forward and backward interpolation?
9. Use Euler's method, find approximate values of $y$ corresponding to $x=1.1$ given that

$$
\frac{d y}{d x}=x+2 y \text { and } y=1 \text { when } x=1
$$

10. Write down the equation for method of least squares with the Normal equations for parabola.

## PART B <br> (Answer one full question from each module, each question carries 14 marks)

## MODULE I

11. a) Suppose that the probabilities are $0.4,0.3,0.2$ and 0.1 , there will be $0,1,2$, or 3 power failures in a certain city during the month of July. Find the mean and variance of this probability distribution.
b) Find the mean and variance of Binomial Distribution.

## OR

12. a) In a factory turning out razor blade, there is a small chance of $\frac{1}{500}$ for any blade to be defective. The blades are supplied in a packet of 10 . Use Poisson distribution to calculate the approximate number of packets containing blades with
i. no defective
ii. one defective
iii. two defectives
in a consignment of 10,000 packets.
b) The joint probability distribution of X and Y is given by
$f(x, y)=\frac{(2 x+y)}{27}, x=0,1,2 ; y=0,1,2$. Find
i. the marginal distributions of X and Y
ii. are $X$ and $Y$ Independent or not.

## MODULE II

13. a) In a Normal Distribution $17 \%$ of the items are below 30 and $17 \%$ of the items are above 60 . Find the mean and standard deviation.
b) The length of time for one individual to be served at a cafeteria is a random variable having an exponential distribution with a mean of 4 minutes. What is the probability that a person is served in less than 3 minutes?

## OR

14. a) If X is uniformly distributed over $(-\alpha, \alpha), \alpha>0$. Find $\alpha$ so that

$$
\begin{equation*}
\mathrm{P}(|\mathrm{X}|<1)=\mathrm{P}(|\mathrm{X}|>1) . \tag{7}
\end{equation*}
$$

b) The joint density function of two continuous random variables X and Y is $\mathrm{f}(\mathrm{x})=\left\{\begin{array}{rc}\text { Cry, } & 0<\mathrm{x}<4,1<\mathrm{y}<5 \\ 0 & , \text { Otherwise }\end{array}\right.$
i. Find the value of the constant C
ii. Find $\mathrm{P}(\mathrm{X} \geq 3, \mathrm{Y} \leq 2)$
iii. Find the marginal distribution of X .

## MODULE III

15. a) A principal at a certain school claim that the students in his school are above average intelligence. A random sample of thirty students IQ scores have a mean_score of 112.5 . Is there sufficient evidence to support the principal's claim? The mean population IQ is 100 with a standard deviation of 15 .
b) In a city 350 out of 600 persons were found to be vegetarian on the basis of this data, can we say that majority if the population in the city is vegetarian?

## OR

16. a) A random sample of 700 units from a large consignment showed that 200 were damaged. Find $95 \%$ and $99 \%$ confidence limits for the proportion of damaged units in the consignment.
b) In a sample of 600 men from a certain city 400 are found to be smokers. In 900 from another city 450 are smokers. Do the data indicates the cities are significantly different as far as smoking habit of people are concerned?

## MODULE IV

17. a) Solve the equation $x e^{x}=2$ by method of False position, correct to four decimal places.
b) Evaluate $\int_{0}^{1} e^{-x^{2}} d x$ by dividing the range of integration in to 4 equal parts using
a) Trapezoidal rule
b) Simpson's rule

## OR

18. a) Using Lagrange's formula find the $y(10)$ to the data

| x | 5 | 6 | 9 | 11 |
| :---: | :---: | :---: | :---: | :---: |
| y | 12 | 13 | 14 | 16 |

b) Find the cubic degree polynomial from the following table using Newton's Divided difference

| x | 0 | 1 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(\mathrm{x})$ | 1 | 4 | 40 | 85 |

## MODULE V

19. a) Solve the system of equation $8 x-y+z-18=0,2 x+5 y-2 z-3=0, x+y-3 z+6=0$ using Gauss- Seidel iterations method.
b) Using Runge - Kutta method of fourth order solve $\frac{d y}{d x}=\frac{y^{2}-x^{2}}{y^{2}+x^{2}}$ with $y(0)=1$ at $x=0.2$.

OR
20. a) Fit a straight line to the following data

| x | 0 | 5 | 10 | 15 | 20 | 25 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| y | 12 | 15 | 17 | 22 | 24 | 30 |

b) Using Adam's method find $y(0.4)$ given $y^{\prime}=\frac{x y}{2} ; y(0.1)=1.01$, $y(0.2)=1.022, y(0.3)=1.023$.

