Register No.: ……………………………… Name: ………………………………………………………….

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FIRST SEMESTER M.TECH DEGREE EXAMINATION (Regular), FEBRUARY 2022

(COMMON TO GEOMECHANICS & STRUCTURES AND STRUCTURAL ENGINEERING & CONSTRUCTION MANAGEMENT)

(2021 Scheme)

Course Code : **21GS101**

Course Name: **Applied Mathematics for Civil Engineers**

Max. Marks : **60** **Duration: 3 Hours**

Non-programmable scientific calculators may be permitted for this examination

PART A

(Answer all questions. Each question carries 3 marks)

1. Show that the Beta function, $\beta(m, n)$ is symmetric.
2. Write the recurrence formula for the Gamma function, $\Gamma(n)$.
3. Show that $y(x) = \int_0^x t(t-x)y(t)dt + 0.5x^2$ is equivalent to the initial value problem, $y'' + xy = 1; y(0) = y'(0) = 0$.
4. Solve the integral equation $\int_0^x y(t)y(x-t)dt = 4\sin(9x)$.
5. Solve the first order partial differential equation, $px + qy = z$.
6. Find the inverse Laplace transform of $log\left(\frac{s^2+1}{s(s+1)}\right)$.
7. Classify the second order partial differential equation, $U_{xx} + 4U_{xy} + (x^2 + 4y^2)U_{yy} = \sin(x + y)$.
8. Form the computational structure for numerical solution of the Laplace's equation and write the Diagonal Five Point Formula.

PART B

(Answer one full question from each module, each question carries 6 marks)

MODULE I

9. Derive the generating function for $J_n(x)$. (6)

OR

10. Prove the relation between the Beta and the Gamma functions. (6)

MODULE II

11. Find the Fourier Transform of $f(x) = \begin{cases} 1; & |x| < 1 \\ 0; & |x| > 1 \end{cases}$. (6)

OR

12. Find the Laplace transform of (i) $\frac{\cos 2t - \cos 3t}{t}$ (ii) $te^{2t}\sin 3t$. (6)

MODULE III

13. Show that the function $y(x) = xe^x$ is a solution of the integral equation $y(x) = \sin(x) + 2\int_0^x \cos(x-t)\, y(t)dt$. (6)

OR

14. Solve the Fredhom integral equation $y(x) = 1 + \lambda\int_0^1 xty(t)dt$. (6)

MODULE IV

15. Solve the Partial differential equation $q + px = p^2$. (6)

OR

16. Solve $p - q = \log(x+y)$. (6)

MODULE V

17. Find a solution of the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2\text{u}$, using the method of separation of variables. (6)

OR

18. A string is stretched and fastened to two points l appart. Motion is started by displacing the string in the form $y = \text{a}\sin\left(\frac{\pi x}{l}\right)$ from which it is released at time t=0. Show that the displacement of any point at a distance x from one end at time t is given by $y(x,t) = \text{a}\sin\left(\frac{\pi x}{l}\right)\cos\left(\frac{\pi ct}{l}\right)$. (6)

MODULE VI

19. Solve the Laplace's equation for the square mesh with boundary values $b_1 = 60, b_2 = 60, b_3 = 60, b_4 = 60, b_5 = 50, b_6 = 40, b_7 = 30, b_8 = 20, b_9 = 10, b_{10} = 0, b_{11} = 20, b_{12} = 40$. (6)

OR

20. Solve numerically the second order partial differential equation, $\nabla^2 u = 0$ with the given boundary conditions $u(0,y) = 0, u(4,y) = 12 + y, 0 \le y \le 4; u(x,0) = 3x, u(x,4) = x^2, 0 \le x \le 4$ with unit displacement in both x and y directions. (6)

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