## 185A1

## SAINTGITS COLLEGE OF ENGINEERING KOTTAYAM, KERALA

(AN AUTONOMOUS COLLEGE AFFILIATED TO
APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)
FIRST SEMESTER M.TECH. DEGREE EXAMINATION (R), MARCH 2021 (TELECOMMUNICATION ENGINEERING)

| Course <br> Code: | 20ECTET103 |
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| Course |  |
| Name: | RANDOM PROCESSES AND APPLICATIONS |
| Max. Marks: | $\mathbf{6 0}$ |

Duration: 3 Hours

## PART A

## (Answer all questions. Each question carries3 marks)

1. Given $\mathrm{f}_{\mathrm{x}}(\mathrm{x})=\mathrm{ke}^{-\theta \mathrm{x}}$ for $\mathrm{x}>0, \theta>0$. Find k .
2. Let X and Y be two independent random variables. IfZ $=\mathrm{X}+\mathrm{Y}$, then find the probability density function of Z
3. Expectation and variance of Poisson random variable
4. Describe Markov chain and transition probability matrix
5. State Central limit theorem
6. State the weak law of large numbers
7. Define ergodic process.
8. StateKarhunen-Loeve expansion

## PART B <br> (Answer one full question from each module, each question carries 6 marks) MODULE I

9. The cumulative distribution function of a random variable X is given by

$$
\mathrm{F}_{\mathrm{x}}(\mathrm{x})=\left\{\begin{array}{c}
0, \mathrm{x}<0 \\
\mathrm{x}^{2}, 0 \leq \mathrm{x}<1 / 2 \\
1-\frac{3}{25}\left(3-\mathrm{x}^{2}\right), 1 / 2 \leq \mathrm{x}<3 \\
1, \mathrm{x} \geq 3
\end{array}\right.
$$

Find the probability density function of X and evaluate $\mathrm{P}(|\mathrm{X}| \leq 1)$ and $\mathrm{P}(1 / 3 \leq \mathrm{X} \leq 4)$ using both the probability density function and cumulative distribution function.

## OR

10. State and prove the addition and multiplication theorems of probability

## MODULE II

11. Two unbiased coins are tossed. Let $x=1$ if the first coin shows head and $x=0$ if it shows tail and let y denote the number of heads thrown. Write down the joint probability density function and the distribution function of ( $\mathrm{x}, \mathrm{y}$ )

## 185A1

OR
12. If X and Y be two independent random variables, compute the probability density function of $\mathrm{Z}=$ $\max (\mathrm{X}, \mathrm{Y})$

## MODULE III

13. Let $\mathrm{X}: \mathrm{N}\left(\mu, \sigma^{2}\right)$.Find moment generating function and hence find first and second moments.

## OR

14. If $f_{X Y}(x, y)=(2-x-y)$ for $0 \leq x \leq 1,0 \leq y \leq 1$, is the joint probability density function of $(X, Y)$, find $E(X+Y)$ and $E(X Y)$. Also verify whether $E(X+Y)=E(X)+E(Y)$ and $E(X Y)=E(X) E(Y)$.

MODULE IV
15. Find the mean, variance, autocovariance and autocorrelation of Poisson process

OR
16. Consider a random process $X(t)=\operatorname{Cos}(t+\varphi)$ where $\varphi$ is a random variable with probability density
function $\quad f_{X}(\varphi)=\frac{1}{\pi}$, where $\frac{-\pi}{2}<\varphi<\frac{\pi}{2}$. Check whether or not the process is stationary.

## MODULE V

17. A random variable $X$ has a mean 12, a variance 9 and an unknown probability distribution. Find

$$
\begin{aligned}
& \mathrm{P}(6<\mathrm{X}<18) \\
& \mathrm{P}(3<\mathrm{X}<21)
\end{aligned}
$$

## OR

18. State and prove Cauchy -Schwartz inequality for random variables

## MODULE VI

19. Let $\mathrm{X}(\mathrm{t})$ be a WSS random process. Define the following for $\mathrm{X}(\mathrm{t})$
(i) Mean -ergodic
(ii) Correlation ergodic
(iii) Mean -square ergodic
(iv) Wide sense ergodic

## OR

20. State and prove the Mean Ergodic Theorem for a random process $X(t)$
