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SAINTGITS COLLEGE OF ENGINEERING
KOTTAYAM, KERALA

(AN AUTONOMOUS COLLEGE AFFILIATED TO
APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FIRST SEMESTER M.TECH. DEGREE EXAMINATION, MARCH 2021
(ROBOTICS & AUTOMATION)

Course Code: 20ECRAT101

Course Name: ADVANCED MATHEMATICS AND OPTIMIZATION TECHNIQUES

Max. Marks: **60** **Duration:** **3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Define basis of a vector space with example.
2. Find the matrix representation with respect to the standard basis in R^2 and the basis D = $\{t^2 + t, t+1, t-1\}$ for the linear transformation T : $R^2 \to R^2$ defined by $T(a, b) = (4a+b)t^2 + 3at + (2a-b)$
3. Find the projection of vector V = (1, −2, 3, −4) along the vector w = (1, 2, 1, 2) in the inner product space R^4 with the standard inner product.
4. Explain the basic assumptions of a linear programming problem
5. Convert the following 0-1 programming problem to standard form

   Maximize : $z = 20x_1 + 40x_2 + 20x_3 + 15x_4 + 30x_5$

   Subject to :

   $$5x_1 + 4x_2 + 3x_3 + 7x_4 + 8x_5 \leq 25$$
   $$x_1 + 7x_2 + 9x_3 + 4x_4 + 6x_5 \leq 25$$
   $$8x_1 + 10x_2 + 2x_3 + x_4 + 10x_5 \leq 25$$
   $$x_i = 0 \text{ or } 1;\ i = 1,2,3,4,5$$

6. Distinguish between integer programming problem and linear programming problem
7. Explain quadratic programming?
8. State Kuhn-Tucker conditions for a nonlinear programming problem having a maximization objective function

PART B

(Answer one full question from each module, each question carries 6 marks)

MODULE I

9. Which of the following subsets of R^3 are subspaces? (6)

   (a) The plane of vectors (b_1, b_2, b_3) with first component $b_1 = 0$

   (b) The vectors (b_1, b_2, b_3) with $b_2 b_3 = 0$

OR

10. Let V be a vector space of 2×2 matrix over R. Let W be a subspace of a symmetric matrix. Find the dimension of W and its basis. (6)

MODULE II

11. Consider the matrix mapping A: $R^4 \rightarrow R^3$ where A = $\begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 3 & 8 & 13 & -3 \end{bmatrix}$ (6)
Find the basis & dimension of image of A & kernel of A.

OR

12. Find transition matrices between the two bases G = {t + 1, t-1} and H= {2t + 1, 3t + 1} for P^1. Verify $V_G = P_H^G V_H$ and $V_H = P_G^H V_G$ for the co-ordinate representation of the polynomial (3t + 5) with respect to each basis (6)

MODULE III

13. Apply Gram-Schmidth orthogonalization process to the basis B = {(1, 1, 1, 1), (1, 2, 4, 5), (1, −3, −4, −2)} of the inner product space R^4 to find orthogonal & orthonormal basis of R^4. (6)

OR

14. Explain least square solution of inconsistent system. Obtain the least square solution of the inconsistent system AX=B where A = $\begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$ (6)

MODULE IV

15. Use Simplex method to solve the following Linear programming problem (6)
Maximize $Z = 3x_1 + 2x_2$
Subject to:

$-x_1 + 2x_2 \le 4$
$3x_1 + 2x_2 \le 14$
$x_1 - x_2 \le 3$
$x_1, x_2, x_3 \ge 0$

OR

16. Find the optimum solution using Big M method (6)
Minimize $Z = 7x_1 + 15x_2 + 20x_3$
Subject to:

$2x_1 + 4x_2 + 6x_3 \ge 24$
$3x_1 + 9x_2 + 6x_3 \ge 30$
$x_1, x_2, x_3 \ge 0$

MODULE V

17. Find the optimum integer solution for the LPP (6)
$Maximize : z = 2x_1 + 3x_2$
$Subject\ to :$

$2x_1 + 2x_2 \le 7$
$x_1 \le 2$
$x_2 \le 2$
$x_1, , x_2 \ge 0$ and integers

OR

18. Solve the LPP using branch and bound method. (6)
$Maximize : z = 5x_1 + 6x_2$
$Subject\ to :$

$x_1 + x_2 \le 5$
$4x_1 + 7x_2 \le 28$
$x_1, x_2 \ge 0$ and integers

MODULE VI

19. Solve the following using Kuhn-Tucker conditions (6)

Maximise $z = x_1^2 + x_1x_2 - 2x_2^2$

Subject to:

$4x_1 + 2x_2 \leq 24$

$5x_1 + 10x_2 \leq 30$

$x_1, x_2 \geq 0$

OR

20. Solve the following nonlinear programming problem (6)

Minimize $z = 2x_1^2 - 3x_2^2 + 18x_2$

Subject to:

$2x_1 + x_2 = 8$

$x_1, x_2 \geq 0$

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