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SAINTGITS COLLEGE OF ENGINEERING
KOTTAYAM, KERALA

(AN AUTONOMOUS COLLEGE AFFILIATED TO
APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

FIRST SEMESTER M.TECH. DEGREE EXAMINATION(R), MARCH 2021
(POWER SYSTEMS)

Course Code: 20EEPST101

Course Name: APPLIED MATHEMATICS

Max. Marks: **60** **Duration:** **3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

1. Find the Z-transform of n^2.
2. Find the shortest smooth plane curve joining two distinct points in the plane.
3. Solve the integral equation $\int_0^x \frac{y(t)}{\sqrt{x-t}}dt = \sqrt{x}$ by transform method.
4. Show that the means of samples taken from a normal population are consistent estimate of the population mean.
5. Explain briefly the natural cubic spline approximation.
6. Derive the normal equations for an exponential $y = ae^{bx}$ using least square error estimate.
7. Let V be the vector space of all 2×2 matrices over the field F. Prove that V has dimension 4 by exhibiting a basis for V which has four elements.
8. Let T and U be the linear operators on R^2 defined by $T(x_1, x_2) = (x_2, x_1)$ and $U(x_1, x_2) = (x_1, 0)$. Find the transformations $U + T, UT, T^2$.

PART B

(Answer one full question from each module, each question carries 6 marks)

MODULE I

9. State and prove convolution theorem. (6)

OR

10. Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = 0, y_1 = 0$ using Z -transforms. (6)

MODULE II

11. Find the extremals of the functional $\int_{x_1}^{x_2} \frac{(1+y^2)}{y'^2}dx$. (6)

OR

12. Determine the shape an absolutely flexible, inextensible homogeneous and heavy rope of given length L suspended at the points A and B. (6)

MODULE III

13. Show that the integral equation $y(x) = \frac{x^3}{6} - x + 1 + \int_0^x \left(\sin t - (x-t)(\cos t + e^t)\right) y(t)dt$ is equivalent to the differential equation $y''(x) - \sin x\, y'(x) + e^x y(x) = x; y(0) = 1, y'(0) = -1.$ (6)

OR

14. Solve the Fredholm integral equation $y(x) = \sin x + \lambda \int_0^{2\pi} \cos(x+t) y(t)dt$ using successive approximation. (6)

MODULE IV

15. If x_1, x_2 and x_3 are three independent observations from a population with mean μ and variance σ^2. Compare the efficiencies of $t_1 = x_1 + x_2 - x_3$ and $t_2 = 2x_1 + 3x_2 - 4x_3$. (6)

OR

16. Show that $t_n = \frac{n\bar{x}}{n+1}$ is a consistent estimate of λ, where $\bar{x}$ is the mean of samples of size n taken from a poisson population. (6)

MODULE V

17. Fit a straight line to the following data, $y(0) = 2.1, y(1) = 3.5, y(2) = 5.4, y(3) = 7.3, y(4) = 8.2$ (6)

OR

18. Determine the constants a, b by the method of least square, such that $y = ae^{bx}$ fits the following values $y(0) = 0.10, y(0.5) = 0.45, y(1) = 2.15, y(1.5) = 9.15, y(2) = 40.35, y(2.5) = 180.75$ (6)

MODULE VI

19. Let V be the vector space of all 2×2 matrices over the field F. Let W_1 be the set of matrices of the form $\begin{bmatrix} x & -x \\ y & z \end{bmatrix}$ and let W_2 be the set of matrices of the form $\begin{bmatrix} a & b \\ -a & c \end{bmatrix}$ (6)
(a) Prove that W_1 and W_2 are subspaces of .
(b) Find the dimensions of $W_1, W_2, W_1 + W_2$ and $W_1 \cap W_2$.

OR

20. Let T be a linear operator on R^3, the matrix of which in the standard ordered basis is $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$. Find a basis for the range of T and a basis for the null space of T. (6)

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