Register No.: .... Name: ....

SAINTGITS COLLEGE OF ENGINEERING (AUTONOMOUS)

(AFFILIATED TO APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY, THIRUVANANTHAPURAM)

SECOND SEMESTER B.TECH DEGREE EXAMINATION (Supplementary), December 2021

Course Code: **20MAT102**

Course Name: **Vector Calculus, Differential Equations and Transforms**

Max. Marks: **100** **Duration: 3 Hours**

PART A

(Answer all questions. Each question carries 3 marks)

		CO
1.	What is the greatest rate of increase of $\phi(x,y,z) = x^2z - 3xy^2z^3$ at (0,1,3).	[1]
2.	A particle moves through 3- space in such a way that its velocity is $\vec{v}(t) = \hat{\imath} + t\hat{\jmath} + t^2\hat{k}$. Find the coordinates of the particle at time $t = 1$, given that the particle is at the point $(-1,2,4)$ at the time $t = 0$.	[1]
3.	Apply Green's theorem to evaluate $\int_c (y^2 - 7y)\, dx + (2xy + 2x)\, dy$ where C is the circle $x^2 + y^2 = 1$.	[2]
4.	Determine whether the vector field $\vec{F}(x,y,z) = (x^3 - x)\hat{\imath} + (y^3 - y)\hat{\jmath} + (z^3 - z)\hat{k}$ is free of sources and sinks. If it is not, locate them.	[2]
5.	Check whether $e^{-0.5x}$, $e^{-2.5x}$ are linearly dependent or independent using Wronskian.	[3]
6.	Given $y_1 = x^2$ is the solution of $x^2y'' + xy'$- 4y = 0, find the second solution by reduction of order.	[3]
7.	Find the Laplace transform of $cos^2\, \omega t$.	[4]
8.	Show that $u(t-a) = \int_0^t \delta(t-a)\, dt$	[4]
9.	Find Fourier cosine integral representing of $f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & x > 1 \end{cases}$	[5]
10.	Find the Fourier sine transform of e^{-ax}, $a > 0$.	[5]

PART B

(Answer one full question from each module, each question carries 14 marks)

MODULE I

			CO	Marks
11.	a)	Find the divergence and the curl of the vector field $\vec{F}(x,y,z) = e^{xy}\hat{\imath} - 2\cos y\hat{\jmath} + sin^2 z\hat{k}$.	[1]	(8)
	b)	Evaluate $\int_c \vec{F} \cdot d\vec{r}$, where C is the path $x = t, y = t^2, z = t^3$ from (0,0,0) to (1,1,1) and $\vec{F} = (3x^2 + 6y)\hat{\imath} - 14yz\hat{\jmath} -\vdash 20xz^2\hat{k}$.	[1]	(6)

OR

			CO	Marks
12.	a)	Determine whether $\vec{F} = (2xy + z^3)\hat{\imath} + x^2\hat{\jmath} + 3xz^2\hat{k}$ is conservative vector field. If so, find the scalar potential. Hence evaluate the work done in moving an object in this field from $(1,-2,1)$ to $(3,1,4)$.	[1]	(8)
	b)	Find the directional derivative of $f(x,y,z) = \frac{y}{x+z}$ at $P(2,1,-1)$ in the direction from P to $Q(-1,2,0)$.	[1]	(6)

MODULE II

			CO	Marks
13.	a)	Evaluate $\iint_\sigma (x+y)dS$, where σ is the portion of the plane $z = 6 - 2x - 4y$ in the first octant.	[2]	(8)
	b)	Find the outward flux of the vector field $\vec{F} = (2x - y)\hat{\imath} + (2y - z)\hat{\jmath} + z^2\hat{k}$ across the surface of the region that is enclosed by the circular cylinder $x^2 + y^2 = 4$ and the plane z= 0 and z= 3.	[2]	(6)

OR

			CO	Marks
14.	a)	Apply Stokes theorem to evaluate the integral $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = 3z\hat{\imath} + 4x\hat{\jmath} + 2y\hat{k}$ and C is the boundary of the paraboloid $x^2 + y^2 + z = 4$ for which $z \geq 0$ and C is positively oriented.	[2]	(8)
	b)	Use Green's theorem to evaluate $\oint_C (e^x + y^2)\, dx + (e^y + x^2)dy$ where C is the boundary of the region between $y = x^2$ and $y = 2x$.	[2]	(6)

MODULE III

			CO	Marks
15.	a)	Using the method of variation of parameter solve the differential equation $(D^2 - 2D + 5)y = e^x tan2x$.	[3]	(8)
	b)	Solve $(D^3 + 3D^2 - 5D - 39I)y = -300cosx$.	[3]	(6)

OR

			CO	Marks
16.	a)	Solve $(D^2 - 4D + 3I)y = e^x - \frac{9}{2}x^2$.	[3]	(8)
	b)	Solve the initial value problem $x^2y'' - 3xy' + 4y = 0 \;\; y(1) = \pi , y'(1) = 4\pi$.	[3]	(6)

MODULE IV

			CO	Marks
17.	a)	Using Laplace transform, solve the differential equation $y'' + 9y = 10e^{-t}, y(0) = 0,\; y'(0) = 0$.	[4]	(8)
	b)	Find $L(te^{-t} \cos t)$.	[4]	(6)

OR

CO Marks

18\. a) Apply convolution theorem to find the Laplace inverse transform of $\frac{240}{(s^2+1)(s^2+25)}$. [4] (8)

b) Determine the response of the damped mass- spring system under a square wave, modeled by [4] (6)

$$y'' + 3y' + 2y = r(t) = u(t-1) - u(t-2),\ y(0) = 0,\ y'(0) = 0$$

MODULE V

CO **Marks**

19\. a) Show that $\int_0^\infty \frac{\sin\omega - \omega\cos\omega}{\omega^2} \sin x\,\omega\, dx \begin{cases} \frac{\pi}{2}x & 0 < x < 1 \\ \frac{\pi}{4} & x = 1 \\ 0 & x > 1 \end{cases}$. [5] (8)

b) Find the Fourier transform of $f(x) = \begin{cases} x & -1 < x < 1 \\ 0 & otherwise \end{cases}$. [5] (6)

OR

CO **Marks**

20\. a) Find the Fourier transform of $f(x) = \begin{cases} 1 & |x| < 1 \\ 0 & |x| > 1 \end{cases}$. [5] (8)

Hence evaluate $\int_0^\infty \frac{\sin t}{t} dt$.

b) Compute the Fourier transform of convolution of the functions [5] (6)

$f(x) = g(x) = e^{-x^2}$.

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