$\qquad$ Name: $\qquad$

# APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY 

Fourth semester B.Tech examinations (S), September 2020

## Course Code: ME202 Course Name: ADVANCED MECHANICS OF SOLIDS (ME)

Data-books are not allowed to use
Max. Marks: 100
Duration: 3 Hours

## PART A

Answer any three full questions, each carries 10marks.
1 a) If the stress tensor at a point is given by $\sigma_{\mathrm{xx}}=0, \sigma_{\mathrm{yy}}=0, \sigma_{\mathrm{zz}}=0, \tau_{\mathrm{xy}}=10, \tau_{\mathrm{xz}}=-10$, $\tau_{y z}=20$, find stress invariants, characteristic equation, principal stresses and the principal plane associated with the maximum principal stress.
b) Find magnitude of normal and shear stress on an octahedral plane for the state of stress specified in part (a) above.
a) Deduce the constitutive relation for a linear elastic isotropic material
b) Derive the compatibility equation in terms of Airy's stress function for a two dimensional elastic problem. Assume body forces are not acting
a) Differentiate plane stress and plane strain problems with suitable examples.
b) Verify whether the function $\phi(\mathrm{x}, \mathrm{y})=\frac{C_{1}}{6}\left(\mathrm{x}^{3}+\mathrm{y}^{3}\right)+\mathrm{C}_{2} \mathrm{xy}$, (C1 and C 2 are constants) is a valid Airy's stress function. If valid, formulate stresses from it.

## PART B

Answer any three full questions, each carries 10marks.
Derive equilibrium equations in (r, $\theta$ ) coordinates; Modify it to axi-symmetric case
6 a) For a 2D axi-symmetric problem, if $\sigma_{\mathrm{r}}=\frac{E}{1-v^{2}}\left[A(1+v)-\frac{B}{r^{2}}(1-v)\right] ; \quad \sigma_{\theta}=$ $\frac{E}{1-v^{2}}\left[A(1+v)+\frac{B}{r^{2}}(1-v)\right]$, evaluate A and B for pressures $\mathrm{P}_{\mathrm{a}}$ and $\mathrm{P}_{\mathrm{b}}$ at inner radius ' $a$ ' and outer radius ' $b$ ' respectively. Then modify $\sigma_{r}$ and $\sigma_{\theta}$ accordingly.
b) Reduce the above equations for a case with external pressure alone. Plot the variation of stresses across the thickness of the wall.

Taking into account only the effect of normal stresses, determine the strain energy of the simply supported beam $A B$ for the loading shown in figure

b) A structure has a vertical leg of length 'L' with the bottom part held fixed. It has a semi-circular part of radius ' $r$ ' as shown and applied by load P. Find deflection at loaded end by Castigliano's method. Consider effects due to bending moment alone.


10 Starting from first principles, demonstrate that the St. Venant's warping function $(\psi)$ should obey the Laplace's equation, $\nabla^{2} \psi=0$.

11 a) Explain the principle of minimum potential energy
b) Using virtual work method, determine the displacement at the end of a cantilever beam of length ' $L$ ' subjected to a uniformly distributed load of w/L.
Compare bending stresses in a $50 \times 50 \mathrm{~mm}^{2}$ Square cross-section bar due to end moments of $2083 \mathrm{~N}-\mathrm{m}$ for (i) straight beam and (ii) curved beam of radius 250 mm (along centroidal curve); moments causing the curve to close.

## PART C

## Answer any four full questions, each carries 10marks.

a) State and explain Maxwell's reciprocal theorem

Using Prandtl's stress function method derive the expression for (i) twist per unit length, (ii) torsional rigidity and (iii) the resultant stress for elliptical cross section under torsion.

Show that the equilibrium conditions for stresses in a membrane loaded due to pressure $(\mathrm{P})$ is analogous to the condition that Prandtl's torsion stress function should obey. Describe the experimental procedure of finding (i) stresses and (ii) torsional rigidity, using the membrane analogy.

14 For the cross-section, if stress due to torsion is limited to $35,0000 \mathrm{KPa}$, find (i) Limiting Torque, (ii) Corresponding Twist per unit length, (iii) Shear stress in web. Use $\mathrm{G}=158 \times 10^{6} \mathrm{kPa}, \mathrm{t}=1.25 \mathrm{~cm}$

(10) (everywhere).

