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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FOURTH SEMESTER B.TECH DEGREE EXAMINATION(S), DECEMBER 2019

Course Code: MA204

Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS

Max. Marks: 100 Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A

Answer any two full questions, each carries 15 marks

1 a) A coin is biased so that the head is twice as likely to appear as the tail. The coin is tossed twice. Find the expected value of the number of heads. Also find the variance of number of heads. (7)

b) Show that Poisson distribution is the limiting case of Binomial distribution as $n \to \infty, p \to 0$. (8)

2 a) The time required to repair a machine is exponentially distributed with parameter $\frac{1}{2}$. What is the probability that (i) repair time exceeds 2 hrs (ii) repair time is between 3 hrs and 5 hrs? (7)

b) A random variable X has the following probability density function: (8)
$f(x) = kx(2-x), 0 < x < 2$. Find (i) the value of k (ii) mean (iii) variance (iv) distribution function.

3 a) Fit a Poisson distribution to the following data: (7)

x	0	1	2	3	4	5
f_i	142	156	69	27	5	1

b) In an intelligence test administrated on 1000 children, the average was 60 and the standard deviation was 20. Assuming that the marks obtained by the children follow a normal distribution, find the number of children who have scored (i) above 90 marks (ii) below 40 marks (iii) between 50 and 80 marks (8)

PART B

Answer any two full questions, each carries 15 marks

4 a) Let $X_1, X_2, \dots \dots, X_{75}$ are independently and identically distributed random variables following Poisson distribution with parameter $\lambda = 2$. Use Central Limit Theorem to estimate $P(120 \le S_{75} \le 160)$, where $S_{75} = X_1 + X_2 + \dots \dots + X_{75}$. (7)

b) The joint density function of two continuous random variables X, Y is given by (8)

$$f(x,y) = \begin{cases} Ke^{-2x-2y}, & x \geq 0, y \geq 0 \\ 0, & otherwise \end{cases}.$$

Find (i) the value of K $(ii) P(X > 1)$ (iii) the marginal distributions of X, Y and (iv) check whether X, Y are independent.

5 a) Find the power spectral density function of the WSS process whose autocorrelation function is $A^2 e^{-2\propto|\tau|}$. (7)

b) Let $X(t) = A\cos(50t + \theta)$, where A and θ are independent random variables. A is a random variable with mean 0 and variance 1 and θ is uniformly distributed in $(-\pi, \pi)$. Show that $\{X(t)\}$ is WSS. (8)

6 a) If $\{X(t)\}$ is a random process with mean 3 and $R(t_1, t_2) = 9 + 4e^{-|t_1 - t_2|/5}$. Find $(i) V[X(5)]$ $(ii) V[X(8)]$ $(iii) Cov\,[x(5), X(8)]$ (7)

b) 3 balls drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. Let X denotes number of white balls drawn and Y denotes number of red balls drawn. Find the joint pdf of X and Y, find the marginal pdfs and check whether X and Y are independent. (8)

PART C

Answer any two full questions, each carries 20 marks

7 a) The tpm of a Markov chain with 3 states 1, 2, 3 is given by $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ with initial distribution $P\{0\} = [0.7, 0.2, 0.1]$. (10)

Find $(i) P\{X_1 = 2/\ X_0 = 1\}$ $(ii)\ P(X_3 = 3, X_2 = 2, X_1 = 1\ \ X_0 = 2)$

$(iii)\ P\{X_2 = 3/X_0 = 1\}$ $(iv)\ P\{X_2 = 3\}$

b) The tpm of a Markov Chain is $P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$. Find the steady state distribution of the chain. (5)

c) Suppose that customers arrive at a bank according to a Poisson process with mean rate of 3 per minute. Find the probability that during a time interval of 2 minutes (5)

(i) exactly 4 customers arrive (ii) more than 4 customers arrive.

8 a) Use Newton's forward interpolation formula to find the interpolating polynomial for the following data. Hence evaluate $y(1.5)$. (7)

x	0	1	2	3
y	0	2	6	18

b) Solve using Runge - Kutta method of order 4: (7)

$y' = 8.5 - 20x + 12x^2 - 2x^3, y(0) = 1$ for $x = 0.5$. [Choose $h = 0.5$]

c) Evaluate $\int_0^2 xe^x dx$ using Simpson's 1/3rd rule with $n = 8$. (6)

9 a) A gambler has rupees 2 and he plays a betting game where he wins Rs. 1, if a tail shows up and loses Rs. 1 if a head shows up in the tossing of a fair coin. He stops playing this game if he wins Rs 2 or loses Rs 2. Find (i) the transition probability matrix of the Markov chain (ii) the probability that the gambler lost his money at the end of 2 plays.(iii) the probability that the gambler ends the game in 2 plays. (10)

b) Using Lagrange's interpolation method to find the value of $f(2)$ from the following data: (5)

x	0	1	3
y	1	3	55

c) Find a positive root of the equation $x^3 + x - 1 = 0$ using Newton-Raphson method correct to 4 decimal places. (5)

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