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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER M.TECH DEGREE EXAMINATION
Electrical and Electronics Engineering
(Power Systems)
04 MA 6303 - APPLIED MATHEMATICS

Time: 3 hrs Max. Marks: 60

PART A

(Answer all questions. Each question carry 3 marks).

1. Find the Fourier transform of $f(x) = e^{-ax}, x > 0, a > 0$. (3)

2. Find the extremals of the functional $\int_{x_0}^{x_1} (x + y')y' dx$ (3)

3. Show that the integral equation $y(x) = \int_0^x (x-t)y(t)dt + 3\sin x$ is equivalent to the differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} + 3\sin x = 0$ where $y = 0$ and $\frac{dy}{dx} = 3$ at $x = 0$. (3)

4. Show that the sample mean is an unbiased estimate of the population mean. (3)

5. Derive the normal equations for a parabolic least square error estimate. (3)

6. Derive Crank-Nicolson formula for the heat . (3)

7. Prove that if two vectors are linearly dependent, one of them is a scalar multiple of the other. (3)

8. Describe explicitly a linear transformation from R^3 into R^3 which has as its range the subspace spanned by $(1, 0, -1)$ and $(1, 2, 2)$. (3)

PART B

(Each full question carries 6 marks).

9. Using the inversion integral method(Residue Method) find the inverse $Z-$ transform of $\frac{10z}{(z-1)(z-2)}$. (6)

OR

10. Find the response of the system $y_{n+2} - 5y_{n+1} + 6y_n = u_n$ with $y_0 = 0, y_1 = 1, u_n = 1$ for $n = 0, 1, 2...$ by $Z-$ transform method. (6)

11. Find the extremal of the functional $\int_0^{\pi} (y'^2 - y^2)dx$ under the conditions $y(0) = 0$, $y(\pi) = 1$ and subject to the constraint $\int_0^{\pi} y dx = 1$. (6)

OR

12. A heavy cable hangs freely under gravity between two fixed points. Show that the shape of the cable is a caternary. (6)

13. Solve $y(x) = x + \frac{1}{6}\int_0^x (x-t)^3 y(t)dt$ by transform method. (6)

OR

14. Solve the Volterra integral equation $y(x) = 2(1+x^2) - \int_0^x xy(t)dt$ using successive approximation. (6)

15. If $\{x_1, x_2, ..., x_n\}$ is a sample of independent observations from a poisson population with parameter λ, obtain an unbiased estimate of $e^{-\lambda}$ (6)

OR

16. Find the maximum likelihood estimates of the mean and variance in the case of a normal population. (6)

17. Fit a least square trend line to the data $y(0) = 5, y(1) = 6, y(2) = 13, y(3) = 32, y(4) = 69$ (6)

OR

18. Determine the constants a and b by the method of least squares such that $y = ae^{bx}$ fits the following data $y(2) = 4.077, y(4) = 11.084, y(6) = 30.128, y(8) = 81.897, y(10) = 222.62$ (6)

19. Show that the vectors $\alpha_1 = (1,0,-1), \alpha_2 = (1,2,1), \alpha_3 = (0,-3,2)$ form a basis for R^3. Express each of the standard basis vectors as linear combination of α_1, α_2 and α_3. (6)

OR

20. Let T be a linear operator on R^3, the matrix of which in the standard ordered basis is (6)

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

Find a basis for the range of T and a basis for the null space of T.