APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M. TECH DEGREE EXAMINATION

Computer Science and System Engineering

04CS6401—Discrete structures for Computer Science

Max. Marks : 60 Duration: 3 Hours

PART A

Answer All Questions

Each question carries 3 marks

1. Let f: R→R, g: R→R defined by f(x) = x^2, g(x) = x+5. Then find fog, gof, fof, gog.

2. Define Hasse diagram. Let A= {1,2,3,4,6,8,12} and R be the partial ordering relation defined by aRb if a divides b. Draw a Hasse diagram for (A,R).

3. Define Tautology. Check whether P→ $(P \vee \sim Q)$ is a Tautology

4. Determine the co-efficient of $x^2y^2z^3$ in the expansion of $(x+y+z)^7$

5. How many arrangements of the letters in MISSISSIPPI have no consecutive S's.

6. A random variable X has a probability distribution p(X=x) = $\begin{cases} c(6-x) & x = 1,2,3,4,5 \\ 0 & otherwise \end{cases}$

   Where c is constant. Find the value of c and hence find the expectation of X.

7. Prove that every group of prime order is cyclic

8. Show that $\begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix}$ is a unit in the ring $M_2(Q)$ but not a unit in $M_2(Z)$ where Q and Z denotes set of all rationals and set of all integers respectively.

PART B

Each question carries 6 marks

9. If f: A→B; $B_1,B_2 \subseteq$ B, then prove the following
   a. $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$
   b. $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$.

OR

10. Let f: A→B, g: B→C, h: C→D , then show that (hog)of = ho(gof)

11. Explain the principal of Mathematical induction. Hence Show that $1^2+2^2+3^2+\ldots\ldots\ldots\ldots\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$ for all natural numbers n.

OR

12. Prove that every equivalence relation defines unique partition of a set.

13. Let $\varphi_1, \varphi_2, \ldots\ldots\ldots \varphi_n$ and ψ be propositional logic formulas. If $\varphi_1, \varphi_2, \ldots\ldots\ldots \varphi_n \vdash \psi$, Then show that $\varphi_1, \varphi_2, \ldots\ldots\ldots \varphi_n \vDash \psi$ holds.

OR

14. Explain completeness of propositional logic.

15. Yosi selects a card from a well shuffled standard deck. What is the probability that his card is a club or a card whose face value is between 3 and 7 inclusive?

OR

16. Box A contains five red and three white marbles and Box B contains four red and six white marbles. If a marble is drawn from each box, what is the probability that they are both of same color.

17. Prove that every sub group of cyclic group G is again cyclic.

OR

18. Let A=$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. Check whether the group G={A,A^2,A^3,A^4} is abelian under matrix multiplication . Is G a cyclic group.

19. State and prove Legrange's theorem on Groups

OR

20. Is the element 777 a unit in Z_{1009}.If so find the multiplicative inverse