

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
**FIRST SEMESTER M. TECH DEGREE EXAMINATION**  
**Electronics & Communication Engineering**  
**(Telecommunication Engineering)**  
**04EC6801—Applied Linear Algebra**

Max. Marks : 60

Duration: 3 Hours

**PART A**

*Answer All Questions*

*Each question carries 3 marks*

1. Express the vector  $v = (1, -2, 5)$  as a linear combination of the vectors  $v_1 = (1, 1, 1)$ ,  $v_2 = (1, 2, 3)$ ,  $v_3 = (2, -1, 1)$ .
2. Explain the row-reduced echelon form.
3. Find the inverse of the linear transformation  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  is defined by  $T(x_1, x_2, x_3) = (2x_1 + 3x_2 + x_3, 3x_1 + 3x_2 + x_3, 2x_1 + 4x_2 + x_3)$ .
4. Find a unit vector orthogonal to  $v_1 = (1, 1, 2)$  and  $v_2 = (0, 1, 3)$  in  $\mathbb{R}^3$ .
5. Explain the properties of eigen values.
6. Determine the eigen values of  $A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 1 & -1 \\ 0 & 2 & -2 \end{bmatrix}$  and their algebraic multiplicity.
7. Explain range and rank of  $AA^T$  and  $A^T A$  with example.
8. Explain Pseudo inverse of a matrix using SVD.

**PART B**

*Each question carries 6 marks*

9. Find the dimension and basis for column space of the matrix  $A = \begin{bmatrix} -1 & 2 & -1 & 5 & 6 \\ 4 & -4 & -4 & -12 & -8 \\ 2 & 0 & -6 & -2 & 4 \\ -3 & 1 & 7 & -2 & 12 \end{bmatrix}$ .

OR

10. a) Show that the vector  $\begin{bmatrix} 20 \\ 4 \end{bmatrix}$  belongs to span of  $\left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix} \right\}$

b) Prove that the vectors  $(1, -1, 1)$ ,  $(0, 1, 2)$  and  $(3, 0, -1)$  form a basis for  $\mathbb{R}^3$ .

11. Find the least square solution to the matrix equation  $\begin{bmatrix} 2 & -2 \\ -2 & 2 \\ 5 & 3 \end{bmatrix} X = \begin{bmatrix} -1 \\ 7 \\ -26 \end{bmatrix}$  using pseudo inverse method.

OR

12. Find the values of ' $\lambda$ ' for which the system of equations  $x + y + z = 1$ ,  $x + 2y + 4z = \lambda$ ,  $x + 4y + 10z = \lambda^2$  will be consistent and also show that for each value of ' $\lambda$ ' the system has a one parameter family of solutions and find these solutions.

13. Find the four fundamental sub spaces of the given matrix  $A = \begin{bmatrix} 1 & -2 & -1 & 3 & 2 \\ 2 & -2 & -3 & 6 & 1 \\ -1 & -4 & 4 & -3 & 7 \end{bmatrix}$ .

OR

14. Given bases of  $\mathbb{R}^2$  :  $S_1 = (u_1 = (1, -2), u_2 = (3, -4))$  and  $S_2 = (v_1 = (1, 3), v_2 = (3, 8))$ . Find the change of basis matrix from  $S_1$  to  $S_2$  and find the change of basis matrix from  $S_2$  back to  $S_1$ .
15. Find the orthonormal basis of the subspace 'W' of  $\mathbb{R}^5$  spanned by  $v_1 = (1, 1, 1, 0, 1)$ ,  $v_2 = (1, 0, 0, -1, 1)$ ,  $v_3 = (3, 1, 1, -2, 3)$ ,  $v_4 = (0, 2, 1, 1, -1)$ .

OR

16. Find a basis for  $W^\perp$  where  $W = \text{span} \left\{ \begin{bmatrix} 1 \\ -5 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ .

17. Compute eigen vectors of the matrix  $A = \begin{bmatrix} 4 & 6 & 10 \\ 3 & 10 & 13 \\ -2 & -6 & -8 \end{bmatrix}$ .

OR

18. Diagonalize the given matrix  $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$ .

19. Find SVD for the matrix  $A = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$ .

OR

20. Find the pseudo inverse of the matrix  $A = \begin{bmatrix} 2 & -2 \\ -1 & 2 \\ 5 & 3 \end{bmatrix}$  using SVD.