# APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY 

FIRST SEMESTER M. TECH DEGREE EXAMINATION<br>MECHANICAL ENGINEERING<br>(MACHINE DESIGN)

04 ME 6501- ADVANCED ENGINEERING MATHEMATICS

## PART A

(Answer all questions. Each question carries $\mathbf{3}$ marks).
1.Write the Euler's equation of the following functional $\int_{0}^{1}\left(y^{2}+12 x y\right) d x$
2. Check whether the following equation is parabolic or not

$$
\begin{equation*}
U_{x x}+4 U_{x y}+4 U_{y y}-U_{x}+2 U_{y}=0 \tag{3}
\end{equation*}
$$

3. Show that $\frac{d}{d x}\left[x^{n} J_{n}(x)\right]=x^{n} J_{n-1}(x)$
4. Classify the equation

$$
\begin{equation*}
y^{2} U_{x x}-2 x y U_{x y}+x^{2} U_{y y}-2 U_{x}-3 U=0 \tag{3}
\end{equation*}
$$

5. Show that velocity of a fluid at any point is a contravariant tensor of rank one
6. Show that
(i) A symmetric tensor of the second order has only $\frac{1}{2} n(n+1)$ different components
(ii) A skew symmetric tensor of the second order has only $\frac{1}{2} n(n-1)$ Different non zero components
7. Describe the various step in ANOVA testing with the rejection region method for One way classifications.
8. Explain the role of the randomization and replication in ANOVA.

## (Each question carries 6 mark. Answer all questions)

9. Show that
(i) $\frac{1}{2}\left[J_{n-1}(x)-J_{n+1}(x)\right]=J_{n}{ }^{\prime}(x)$
(ii) $\frac{n}{x}\left[J_{n}(x)\right]-J_{n+1}(x)=J_{n}{ }^{\prime}(x)$

OR
10. Solve the equation in series $x y^{\prime \prime}+2 y^{\prime}+x y=0$
11. Prove that the shortest distance between any two points in a plane is a straight line (6)

OR
12. Find the extremal of the functional $\int_{0}^{1} \frac{y^{2}}{x^{3}} d x$
13. Prove the orthogonality of Legendre polynomials.

OR
14 Solve in series $y^{\prime \prime}+x y=0$
15. Solve by Crank-Nicholson method $U_{x x}=U_{t}$ subjected to the condition

$$
\begin{equation*}
U(x, 0)=0, U(0, t)=0, U(1, t)=t \quad \text { for two time steps } \tag{6}
\end{equation*}
$$

OR
16. Evaluate the pivotal values of the equation $U_{x x}=16 U_{x x}$ taking $\mathrm{h}=1$, upto $\mathrm{t}=1.25$ The boundary conditions are $U_{t}(x, 0)=0, U(x, 0)=x^{2}(5-x)$
17. Find the components of the metric tensor and the conjugate tensor in the cylindrical coordinates.

## OR

18. (i) Show that any inner product of the tensor $A_{r}^{p}$ and $B_{t}^{q S}$ is a tensor of rank 3
(ii) A covariant tensor has components $2 x-z, x^{2} y, y z$ in the Cartesian coordinates.

Find its components in cylindrical coordinates
19. Following are the weekly sale records (in thousand Rs) of the three sales men A, B, C of a company during 13 sale calls.

| A | 300 | 400 | 300 | 500 |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| B | 600 | 300 | 300 | 400 |  |
| C | 700 | 300 | 400 | 600 | 500 |

Test whether the sales of 3 salesmen are different.

OR
20. The following are the defective pieces produced by four operators working in turn on four different machines.

| Machine | Operator |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | B1 | B2 | B3 | B4 |
| A1 | 34 | 28 | 33 | 29 |
| A2 | 31 | 24 | 35 | 22 |
| A3 | 27 | 20 | 43 | 72 |
| A4 | 28 | 28 | 29 | 26 |

