

| 5 | a) | a) $\operatorname{Rot}(z, 90) \cdot \operatorname{Rot}(x, 90) . \operatorname{Trans}(0,0,3) \operatorname{Trans}(0,5,0)(2$ marks $)$ <br> b) $\left[\begin{array}{cccc}0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1\end{array}\right]\left[\begin{array}{cccc}1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1\end{array}\right]\left[\begin{array}{llll}1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1\end{array}\right]\left[\begin{array}{llll}1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1\end{array}\right]\left[\begin{array}{c}1 \\ 5 \\ 4 \\ 1\end{array}\right]=\left[\begin{array}{c}7 \\ 1 \\ 10 \\ 1\end{array}\right]$ <br> ( 4 marks for 4 transformation matrices and 2 marks for final answer) |
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|  | b) | Matrix representing the orientation change with Euler angles (4 marks) $\begin{aligned} & \operatorname{Euler}(\phi, \theta, \psi)=\operatorname{Rot}(a, \phi) \operatorname{Rot}(o, \theta), \operatorname{Rot}(a, \psi) \\ & \quad=\left[\begin{array}{cccc} C \phi C \theta C \psi-S \phi S \psi & -C \phi C \theta S \psi-S \phi C \psi & C \phi S \theta & 0 \\ S \phi C \theta C \psi+C \phi S \psi & -S \phi C \theta S \psi+C \phi C \psi & S \phi S \theta & 0 \\ -S \theta C \psi & S \theta S \psi & C \theta & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \end{aligned}$ <br> Three fundamental rotations involved in this (3 marks for three fundamental rotations) <br> Rotation of $\phi$ about the $a$-axis ( $z$-axis of the moving frame) followed by, Rotation of $\theta$ about the $o$-axis ( $\gamma$-axis of the moving frame) followed by, Rotation of $\psi$ about the $a$-axis ( $z$-axis of the moving frame). |
| 6 | a) | Asssigning frames (1 mark) <br> Parameter table (2 marks) <br> Writing $A_{1}$ and $A_{2}$ (2 marks) <br> Final transformation matrix ( $\mathbf{3}$ marks) $\begin{aligned} & A_{1}=\left[\begin{array}{cccc} C_{1} & -S_{1} & 0 & a_{1} C_{1} \\ S_{1} & C_{1} & 0 & a_{1} S_{1} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \text { and } A_{2}=\left[\begin{array}{cccc} C_{2} & -S_{2} & 0 & a_{2} C_{2} \\ S_{2} & C_{2} & 0 & a_{2} S_{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \\ & { }^{0} T_{H}=A_{1} \times A_{2}=\left[\begin{array}{cccc} C_{1} C_{2}-S_{1} S_{2} & -C_{1} S_{2}-S_{1} C_{2} & 0 & a_{2}\left(C_{1} C_{2}-S_{1} S_{2}\right)+a_{1} C_{1} \\ S_{1} C_{2}+C_{1} S_{2} & -S_{1} S_{2}+C_{1} C_{2} & 0 & a_{2}\left(S_{1} C_{2}+C_{1} S_{2}\right)+a_{1} S_{1} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \end{aligned}$ |



The kinetic energy of the system is comprised of the kinetic energies of the cart and the pendulum. Notice that the velocity of the pendulum is the summation of the velocity of the cart and of the pendulum relative to the cart, or:

$$
\mathbf{v}_{p}=\mathbf{v}_{c}+\mathbf{v}_{p / c}=(\dot{x}) \mathbf{i}+(\dot{\theta} \cos \theta) \mathbf{i}+(\dot{\theta} \sin \theta) \mathbf{j}=(\dot{x}+\dot{\theta} \cos \theta) \mathbf{i}+(\dot{\theta} \sin \theta) \mathbf{j}
$$

and $v_{p}^{2}=(\dot{x}+\dot{\theta} \cos \theta)^{2}+(\dot{\theta} \sin \theta)^{2}$
Therefore:

$$
\begin{aligned}
K & =K_{\text {cart }}+K_{\text {pendulum }} \\
K_{\text {cart }} & =\frac{1}{2} m_{1} \dot{x}^{2} \\
K_{\text {pendulum }} & =\frac{1}{2} m_{2}\left((\dot{x}+\dot{l} \cos \theta)^{2}+(\dot{\theta} \sin \theta)^{2}\right) \\
K & =\frac{1}{2}\left(m_{1}+m_{2}\right) \dot{x}^{2}+\frac{1}{2} m_{2}\left(l^{2} \dot{\theta}^{2}+2 \dot{\theta} \dot{x} \cos \theta\right)
\end{aligned}
$$

Likewise, the potential energy is the summation of the potential energies in the spring and in the pendulum, or:

$$
P=\frac{1}{2} k x^{2}+m_{2} g l(1-\cos \theta)
$$

Notice that the zero potential energy line (datum) is chosen at $\theta=0^{\circ}$. The Lagrangian will be:
$L=K-P=\frac{1}{2}\left(m_{1}+m_{2}\right) \dot{x}^{2}+\frac{1}{2} m_{2}\left(l^{2} \dot{\theta}^{2}+2 l \dot{\theta} \dot{x} \cos \theta\right)-\frac{1}{2} k x^{2}-m_{2} g l(1-\cos \theta)$

|  |  | The derivatives and the equation of motion related to the linear motion will be: $\begin{aligned} \frac{\partial L}{\partial \dot{x}} & =\left(m_{1}+m_{2}\right) \dot{x}+m_{2} l \dot{\theta} \cos \theta \\ \frac{d}{d t}\left(\frac{\partial L}{\partial \dot{x}}\right) & =\left(m_{1}+m_{2}\right) \ddot{x}+m_{2} l \ddot{\theta} \cos \theta-m_{2} \dot{\theta}^{2} \sin \theta \\ \frac{\partial L}{\partial x} & =-k x \\ F & =\left(m_{1}+m_{2}\right) \ddot{x}+m_{2} l \ddot{\theta} \cos \theta-m_{2} l \dot{\theta}^{2} \sin \theta+k x \end{aligned}$ <br> and for the rotational motion, it will be: $\begin{aligned} \frac{\partial L}{\partial \dot{\theta}} & =m_{2} l^{2} \dot{\theta}+m_{2} l \dot{x} \cos \theta \\ \frac{d}{d t}\left(\frac{\partial L}{\partial \dot{\theta}}\right) & =m_{2} l^{2} \ddot{\theta}+m_{2} l \ddot{x} \cos \theta-m_{2} l \dot{x} \dot{\theta} \sin \theta \\ \frac{\partial L}{\partial \theta} & =-m_{2} g l \sin \theta-m_{2} l \dot{\theta} \dot{x} \sin \theta \\ T & =m_{2} l^{2} \ddot{\theta}+m_{2} l \ddot{x} \cos \theta+m_{2} g l \sin \theta \end{aligned}$ <br> If we write the two equations of motion in matrix form, we will get: $\begin{aligned} F & =\left(m_{1}+m_{2}\right) \ddot{x}+m_{2} l \ddot{\theta} \cos \theta-m_{2} \dot{\theta}^{2} \sin \theta+k x \\ T & =m_{2} l^{2} \ddot{\theta}+m_{2} l \ddot{x} \cos \theta+m_{2} g l \sin \theta \\ {\left[\begin{array}{c} F \\ T \end{array}\right] } & =\left[\begin{array}{cc} m_{1}+m_{2} & m_{2} l \cos \theta \\ m_{2} l \cos \theta & m_{2} l^{2} \end{array}\right]\left[\begin{array}{c} \ddot{x} \\ \ddot{\theta} \end{array}\right]+\left[\begin{array}{cc} 0 & -m_{2} l \sin \theta \\ 0 & 0 \end{array}\right]\left[\begin{array}{l} \dot{x}^{2} \\ \dot{\theta}^{2} \end{array}\right]+\left[\begin{array}{c} k x \\ m_{2} g l \sin \theta \end{array}\right] \end{aligned}$ |  |
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|  | b) | Explain the structure of robot programming language. |  |
| 8 | a) | Jacobian operator and explanation (2 marks) linear velocity Jv of end-effector (4 marks) and angular velocity Jw of end-effector (4 marks) |  |
|  | b) | Textual programming ( 5 marks) Lead through programming (5 marks) |  |
| 9 | a) | Write VAL commands for controlling end-effector motion of a robot. |  |
|  | b) | What is the role of inverse Jacobian operator (5 marks) Significance of singularities (5 marks) |  |

