EC202 – Signals and Systems

(Please note the following regarding the scheme of valuation)

Question No	**Correction**
2a)	Error in mark distribution. 3 marks may be given for diagrams illustrating different shifted positions of h(-t).
2b)	Error in mark distribution. 2 marks may be given for forming matrices,2 marks for obtaining convolution and 2 marks for marking zeroth position correctly.
3 a)	Arrow mark shown incorrectly.So 4 marks may be given for finding odd and even components if done correctly
4a)	can be exponential Fourier Series also: Cn = A/2 for n=0 Cn = $-A/(j2n\pi)$ for $n \neq 0$
7 b ii)	the factor 1/2 to be taken
7b i)	Total mark as per qp is 3 marks. Therefore mark may be given as 2+1
7b ii)	Total mark as per qp is 5 marks. Therefore mark may be distributed accordingly

SET-2

Answer key.

1a

$x(n) = \{ \underset{\uparrow}{2}, 1, 2, 1, 1, 3 \}$

(i) $x(n+1) \rightarrow \{ 2, \underset{\uparrow}{1}, 2, 1, 1, 3 \}$

$x(2n+1) \rightarrow \{ \underset{\uparrow}{1}, 1, 3 \}$ → 1 Mark

$x(-2n+1) \rightarrow \{ 3, 1, \underset{\uparrow}{1} \}$ → 1 Mark

$2x(-2n+1) \rightarrow \{ 6, 2, \underset{\uparrow}{2} \}$ → 1 Mark

---

(ii) $-x\left(\frac{n}{2}-2\right)$

$x(n-2) \rightarrow \{ \underset{\uparrow}{0}, 0, 2, 1, 2, 1, 1, 3 \}$

$x\left(\frac{n}{2}-2\right) \rightarrow \{ \underset{\uparrow}{0}, 0, 0, 0, 2, 0, 1, 0, 2, 0, 1, 0, 1, 0, 3, 0 \}$

$-x\left(\frac{n}{2}-2\right) \rightarrow \{ 0, 0, 0, 0, -2, 0, -1, 0, -2, 0, -1, 0, -1, 0, -3 \}$

→ 1 Mark for each Step.

$\boxed{1b}$

(i)

$$x(n) = (1/3)^n u(n)$$

$$E = \lim_{N\to\infty} \sum_{n=-N}^{N} \left[(1/3)^n\right]^2 u(n)$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{9}\right) = \frac{9}{8}$$

1 Mark

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{9}\right)^n$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left[\frac{1-(1/9)^{N+1}}{1-1/9}\right]$$

1 Mark

$$= 0$$

Energy is finite, Power is 0
So it is energy signal.

1 Mark

(ii)

$$x(t) = (1+e^{-5t})\,u(t)$$

$$E = \lim_{T\to\infty} \int_0^T (1+e^{-5t})^2\, dt$$

$$= \lim_{T\to\infty} \int_0^T 1+e^{-10t}+2e^{-5t}\, dt$$

$$= \infty$$

→ 1 Mark

page 3

$$P = \lim_{T \to \alpha} \frac{1}{2T} \int_0^T 1 + e^{-10t} + 2e^{-5t} \, dt$$

$$= \lim_{T \to \alpha} \frac{1}{2T} \left[T - \frac{1}{10} e^{-10t} \Big|_0^T - \frac{2}{5} e^{-5t} \Big|_0^T \right]$$

$$= \underline{\underline{\frac{1}{2} W}}$$

→ 1Mark.

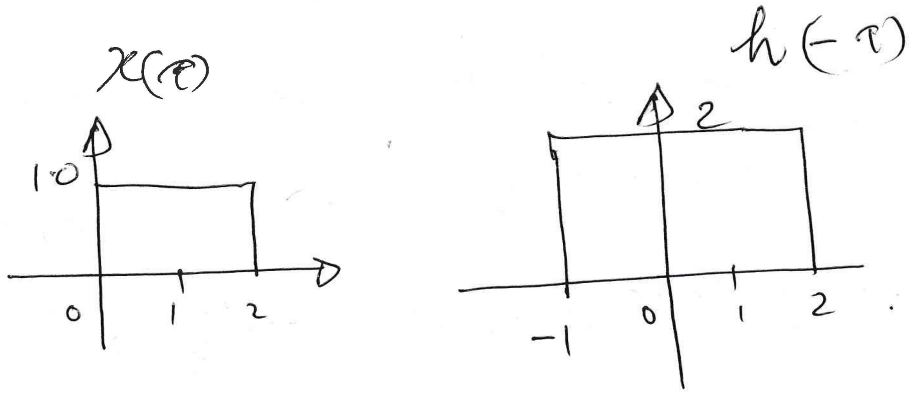
Energy is infinite, power is finite
Signal is a power signal.

→ 1Mark.

$\boxed{1c}$

Define δ function → ½ mark
Sketch δ function → ½ mark.
List properties → 2 Marks.

2a



→ 1 mark.

Diagrams illustrating different shifted positions of $h(-\tau)$ → 2 marks.

Finding the convolution

$y(t) = 0 \quad t > 3$

$y(t) = 0 \quad t \leq -2$

$= 2t + 4 \quad$ for $-2 \leq t < 0$

$= 4 \quad$ for $0 \leq t < 1$

$= 6 - 2t \quad$ for $1 \leq t < 3$

$= 0 \quad$ for $t \geq 3$.

→ 4 marks.

Sketch convolution result → 1 mark.

$\boxed{2b}$ Forming the matrices → 1 mark.

Obtaining Convolution → 2 marks.

Marking 0th position correctly → 1 mark.

$$z(n) = \{2, -7, \underset{\uparrow}{10}, -3, -9, -2\}.$$

$\boxed{2c}$ List 6 properties of discrete time Convolution. ½ mark for each.

$\boxed{3a}$

$$x(t) = x_e(t) + x_o(t)$$

where $x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$

$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

$$x(n) = x_e(n) + x_o(n)$$

$$x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$$

$$x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$$

Derivation for Continuous Case → 2 Mark
Derivation for discrete Case → 1 Mark.

$$x(n) = \{-2, 1, \underset{\uparrow}{2}, -1, 3\}$$

$$\left.\begin{aligned} x_e(n) &= \{0.5, 0, 2, 0, 0.5\}. \\ x_o(n) &= \{-2.5, 1, 0, -1, 2, 5\}. \end{aligned}\right\} \text{4 marks.}$$

page 7

$\boxed{3b}$

$$x(n) = \left(\tfrac{1}{3}\right)^{-n} u(-n-1)$$
$$h(n) = u(n-1)$$

Find the convolution ~~for~~ of $x(n)$ & $h(n)$

Identify the limits of summation for different cases $\rightarrow$ 2 Marks.

$$y(n) = \sum_{k=-\infty}^{-1} 3^k \quad \text{for } n > 0 \quad = 0.5$$

$\rightarrow$ 2 Marks.

$$y(n) = \sum_{k=-\infty}^{n-1} 3^k \quad \text{for } n \leq 0. \quad = 0.5\,(3)^n$$

$\rightarrow$ 2 Marks.

Plotting the result $\rightarrow$ 2 Marks.

4a Forming the function $x(t)$. → 1 mark.

Finding $a_0 = A/2$, $a_n = 0$ } 2 Marks

$b_n = \frac{-A}{n\pi}$ } 2 Marks.

Express $x(t)$ as Fourier Series.

$$x(t) = \frac{A}{2} - \frac{A \sin \omega_0 t}{\pi} - \frac{A \sin 2\omega_0 t}{2\pi} - \cdots$$

→ 2 Marks.

Plotting magnitude spectrum → 1 Mark.

4b $x(t) = t e^{-at}$

$$e^{-at} u(t) \leftrightarrow \frac{1}{a+j\omega}$$

Use $FT\{t\,x(t)\} = j \frac{d}{d\omega} X(j\omega)$

(State and prove this property) ⇒ 4 Mark

$$F\{t e^{-at} u(t)\} = \frac{1}{(a+j\omega)^2}$$ ⇒ 3 Mark.

$\boxed{15a}$

$$H(s) = \frac{1}{(s+1)(s+0.5)}$$

$$x(t) = u(t), \quad X(s) = \frac{1}{s}$$

$$Y(s) = H(s) \cdot X(s)$$

$$= \frac{1}{s(s+0.5)(s+1)}$$

Partial fraction expansion

$$Y(s) = \underbrace{\frac{2}{s}}_{\text{Due to pole of i/p}} + \underbrace{\frac{2}{s+1} - \frac{4}{s+0.5}}_{\text{due to poles of transfer function}}$$

3 Marks.

$$y(t) = \underbrace{2}_{\text{Steady state response}} + \underbrace{2e^{-t} - 4e^{-0.5t}}_{\text{Transient response}}$$

2 Marks.

$\boxed{5b}$ $$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t).$$

Find $H(j\omega) = \dfrac{2}{(j\omega)^2 + 6(j\omega) + 8}$ → 2 Marks.

Find inverse FT to find $h(t)$.

Split $H(j\omega)$ in to partial fractions.

$$H(j\omega) = \frac{-1}{j\omega+4} + \frac{1}{j\omega+2}$$ → 2 Marks

$$h(t) = -e^{-4t}u(t) + e^{-2t}u(t)$$ → 2 Marks.

$\boxed{5c}$ $$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$$

when initial state is zero

$$y(t) = \int_{0}^{t} x(\tau)\,d\tau.$$

} 2 Marks

Apply Laplace Transform $Y(s) = \dfrac{X(s)}{s}$

$$H(s) = \frac{1}{s}$$

} 2 Marks.

$\boxed{6a}$

$$X(s) = \frac{2s+1}{(s+1)(s^2+2s+2)}$$

Split in to partial fractions.

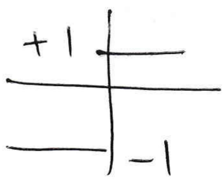
$$X(s) = \frac{2}{s+2} + \frac{1}{s+1} + \frac{1}{(s+1)^2}$$ → 3 Marks.

$$x(t) = \mathcal{L}^{-1}\{X(s)\}$$

$$= 2e^{-2t}u(t) + e^{-t}u(t) + te^{-t}u(t).$$

→ 2 Marks.

$\boxed{6b}$



~~Let~~ $\mathrm{Sgn}(t) = 2u(t) - 1 = x(t)$.

$$\frac{dx(t)}{dt} = \frac{d}{dt}\left[2u(t) - 1\right] = 2\delta(t).$$

(i) Take FT

$$j\omega \cdot X(\omega) = 2$$

$$X(\omega) = \frac{2}{j\omega}$$

→ 2½ Marks.

(ii) $u(t) = \frac{1}{2} + \frac{1}{2}\,\mathrm{Sgn}(t)$.

$FT\{1/2\} = \frac{1}{2} \cdot 2\pi\delta(\omega) = \pi\delta(\omega)$.

$FT\{1/2\,\mathrm{Sgn}(t)\} = \frac{1}{2} \cdot \frac{2}{j\omega} = \frac{1}{j\omega}$

$FT\{u(t)\} = \pi\delta(\omega) + \frac{1}{j\omega}$

→ 2½ Marks.

6c State and prove Parsevals Energy theorem applicable to CTFT.

Statement of theorem/equation → 2 Marks

Proof → 3 Marks.

7a $x(n) = \sin\left(\frac{\pi n}{2}\right) u(n)$.

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x(n) e^{-jwn}$$

$$= \sum_{n=0}^{\infty} \left[\frac{e^{j\pi n/2} - e^{-j\pi n/2}}{2j}\right] e^{-jwn}$$

$$= \frac{1}{2j}\left[\frac{1}{1-e^{j\pi/2}e^{-jw}} - \frac{1}{1-e^{-j\pi/2}e^{-jw}}\right]$$

→ 5 marks.

Re arrange the above equation to get

$$\cancel{X}(e^{jw}) = \frac{e^{-jw}}{1+e^{-j2w}} \quad \rightarrow 3 \text{ marks.}$$

$\boxed{7b}$ (i) $x(n) = a^{|n|}$; $|a| < 1$

$$X(z) = \sum_{n=-\infty}^{\infty} a^{|n|} z^{-n}$$

$$= \sum_{n=0}^{\infty} (az)^n + \sum_{n=0}^{\infty} (az^{-1})^n \rightarrow 1 \text{ Mark.}$$

$$= \frac{az}{1-az} + \frac{1}{1-az^{-1}} \rightarrow 2 \text{ Marks}$$

ROC: $|a| < |z| < \frac{1}{|a|}$ $\rightarrow$ 1 mark

$\boxed{7b}$ (ii) $y(n) = \frac{1}{2} n^2 (1/3)^{n-1} u(n-1)$.

$$Z\{(1/3)^n u(n)\} = \frac{z}{z - 1/3}$$

$$Z\{(1/3)^{n-1} u(n-1)\} = \frac{1}{z-1/3} \rightarrow 1 \text{ Mark.}$$

$$Z\{n(1/3)^{n-1} u(n-1)\} = -z \frac{d}{dz}\left(\frac{1}{z-\frac{1}{3}}\right) = \frac{z}{(z-1/3)^2}$$

$$Z\{n^2 (1/3)^{n-1} u(n-1)\} = -z \frac{d}{dz}\left\{\frac{z}{(z-1/3)^2}\right\} = \frac{z(z+1/3)}{(z-1/3)^3}$$

$\rightarrow$ 2 Marks.

ROC: $\rightarrow$ 1 Mark.. ROC $|z| > 1/3$.

$\boxed{7c}$ Convolution theorem – 1 Mark

Proof – 3 Marks.

$\boxed{8a}$ $y(n) - \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) = x(n)$.

Taking DTFT and getting Transfer function

$$H(e^{j\omega}) = \frac{1}{1 - \frac{1}{6}e^{-j\omega} - \frac{1}{6}e^{-j2\omega}} \rightarrow 3 \text{ Marks.}$$

$$h(n) = F^{-1}\{H(e^{j\omega})\}$$

Breaking in to partial fractions

$$H(e^{j\omega}) = \frac{3}{5} \cdot \frac{1}{1 - \frac{1}{2}e^{-j\omega}} + \frac{2}{5} \cdot \frac{1}{1 + \frac{1}{3}e^{-j\omega}}$$

$\rightarrow$ 3 Marks.

$$h(n) = \frac{3}{5}\left(\frac{1}{2}\right)^n u(n) + \frac{2}{5}\left(\frac{-1}{3}\right)^n u(n).$$

$\rightarrow$ 2 Marks.

8b $y(n) = y(n-1) - 0.5\,y(n-2) + x(n) + x(n-1)$.

Find out $H(z)$, transfer function in z domain

$$H(z) = \frac{z(z+1)}{(z-0.5-j0.5)(z-0.5+j0.5)}$$

→ 4 Marks..

Zeros are $z=0, \; z=-1$

Poles are $0.5+j0.5$, $0.5-j0.5$ → 2 Marks.

plot the pattern. → 1 Mark.

Assess the stability : Stable → 1 Mark.

8c $X(z) = \dfrac{z}{(z-0.2)(z+0.9)}$

replace $z \rightarrow e^{jw}$

$H(e^{jw}) = H(z)\big|_{z=e^{jw}}$ → 1 Mark

$= \dfrac{e^{jw}}{(e^{jw}-0.2)(e^{jw}+0.9)}$ → 3 Marks.

$\boxed{9a}$ $h(n) = (1/2)^n u(n)$

$x(n) = (3/4)^n u(n).$

$$H(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}} \longrightarrow 1\text{ Mark}$$

$$X(e^{j\omega}) = \frac{1}{1-\frac{3}{4}e^{-j\omega}} \longrightarrow 1\text{ Mark}$$

$$Y(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega}).$$

$$= \frac{e^{j\omega}}{e^{j\omega}-3/4} \cdot \frac{e^{j\omega}}{e^{j\omega}-1/2} \rightarrow 1\text{ Mark.}$$

$$\frac{Y(e^{j\omega})}{e^{j\omega}} = \frac{e^{j\omega}}{(e^{j\omega}-3/4)(e^{j\omega}-1/2)}.$$

By Partial fraction

$$Y(e^{j\omega}) = \frac{3}{1-\frac{3}{4}e^{-j\omega}} - \frac{2}{1-\frac{1}{2}e^{-j\omega}} \rightarrow 3\text{ Mark}$$

$$y(n) = \left[3\,(3/4)^n - 2\,(1/2)^n\right] u(n). \rightarrow 2\text{ Mark}$$

$\boxed{9b}$

$x(n) = a^n u(n) - b^n u(-n-1)$

$X(z) = \sum_{n=0}^{\infty} (az^{-1})^n + \sum_{n=1}^{\infty} (b^{-1}z)^n$ → 2 Marks.

$= \frac{z}{z-a} + \frac{z}{z-b}$. → 2 Marks

ROC: $|a| < |z| < |b|$ → 2 Marks.

Plotting the ROC → 2 Marks.

$\boxed{9c}$

Plot the S plane, z plane. → 2 Marks

Mark regions and show the Correspondance. → 2 Marks.